MARYLAND INSTITUTE HANDBOOK

Written and Compiled for the Use
of the Students by

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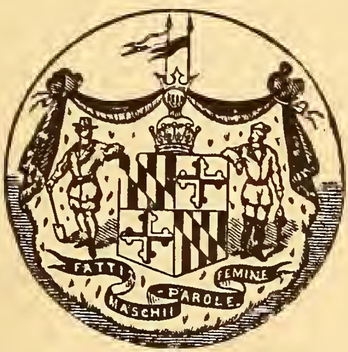

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Dedicated to
HENRY ADAMS, M. E.,
Chairman of Committee, Market Place School,
in appreciation of his untiring efforts
in promoting the interests of
the School

PREFACE

The Maryland Institute Schools of Art and Design, of Baltimore, Maryland (NIGHT SCHOOLS) provides a course in Mechanical Drawing, the students of which vary in age from 16 to 50 years, and are recruited from all classes of workers: shopmen, clerks, apprentices, and young men still attending day schools. These find that in their work a knowledge of mechanical drawing and some knowledge of mechanics are essential. Many, due to lack of application, have forgotten much of that learned at school, while the majority have attended only the lower grades of the common schools.

The object of the following pages is, in so far as possible, to help these students. The course, however, being primarily for teaching mechanical drawing and relatively short, the time allowable for mechanics is very limited. In view of this, only the very elementary principles are considered, and then only to a point from which, coupled with the lectures, problems, etc., the student may further aid himself by use of more advanced text and handbooks, afterward referred to.

In the matter of the strength of materials, the meaning of the various technical terms generally used and their relationship one to another are explained; also the application of simple formulas are shown. These with the lectures, examples and problems by the instructor. should so fit the student that he may open and use a handbook without the usual fear and anxiety experienced upon entering an unknown field.

It is earnestly hoped the student will follow up the work by further study, for nothing can be gained without effort. (So keep working.)

The more knowledge you have of a subject, the more interesting it becomes. (So seek.)

What man has done, man can do again. (Do also.)

As man is imperfect, so, too, are machines. (Improve yourself and try to improve that with which you have to deal.)

CHAPTER I

DEFINITIONS

MECHANICS

This word is used in two ways:

One to indicate that class of skilled workers, machinists, carpenters, bricklayers, etc., who use tools and implements to produce new machines or objects.

The *other* and broader meaning—the "science of forces and powers, their action and application."

MACHINE

This term is also used in two ways:

One to indicate all tools from the simplest to the most intricate machine.

The *other* to all apparatus used to modify or regulate the effects of natural forces. In either case the machine may be simple or complex, depending upon the work it has to do, whether it be to modify a force or modify a motion.

FORCE

Any cause which produces or tends to produce motion or a change of motion.

Force is invisible and is known only by the effect produced.

GRAVITY

One force with which man has continually to deal.

This natural force—like all other natural forces—acts always in the same way under the same circumstances.

It is the attraction between all objects and the earth. Its effect is shown by causing a body to have weight. It acts always in a straight line and toward the center of the earth.

An object held in the hand, i. e., a glass, book, or piece of iron, is said to have weight, and must be supported to counteract the force of gravity to prevent it from falling to the ground, and in a straight line at that.

CENTER OF GRAVITY

A point within the object from which its entire weight may be suspended or supported by a single vertical force.

Whenever any object is suspended by a single cord attached thereto, the center of gravity will lie somewhere in the body in line with the cord.

Forces other than gravity are met with in mechanics, either singly or in groups and in direction other than toward the center of the earth.

ACTION AND REACTION

To every action there is a reaction of like intensity and in the opposite direction. It takes as much force pushing upward to sustain the glass, book, or iron as they push downward when held in the hand. If these two forces were unequal, the object would move in the direction of the greater one. If the object be suspended by a cord, the reaction would be through the cord, and for its entire length.

Forces are also known as Internal or External—the former acting in the object tending to change its form, and the latter acting from the outside, either tending to change its position called "Pressure" or actually moving the object called "Moving Force."

Internal forces may be active or reactive—active when the particles of matter tend to separate, as in steam and gases; reactive when resisting destruction against forces externally applied.

Magnetism acts between the particles of matter forming the body, and therefore is classed as an internal force.

EQUILIBRIUM

When two forces act one against the other and no motion is produced, the forces are in equilibrium or balanced.

Forces being invisible, we know them only by the effects they produce, and then only when we can measure the intensity and know the direction in which they act and the point on which they act. The intensity is measured by a weight or its equivalent. The direction is opposite to that in which the counteracting force acts.

In this country the unit of measurement is the pound or the ton (2,000 lbs.)—depending whether the force be small or large.

RESULTANT

Two forces or more acting on a single point may be combined, so that one counteracting force may hold them in check. Such combined force is called the Resultant force.

If the two or more forces act in the same direction and in the same line, the resultant is equal to the sum of the forces and equal to the counteracting force acting in the opposite direction.

If two forces of different intensity act on the same point but in opposite directions, the resultant is equal to the difference between the two forces, and acts in the same

direction as the greater one and is equal to the counterbalancing force acting in the opposite direction.

When two forces act upon the same point other than in a straight line, i. e., at an angle to each other, the resultant is neither equal to the sum nor the difference between the forces, nor in the direction of either, but a combination of both.

Two methods are used to find the resultant, either by trigonometrical formula or by graphic representation.

The latter is the more convenient, and is frequently referred to as the parallelogram of forces and force diagram.

FORCE DIAGRAM

A force can be measured, and a line may represent it by its length and direction, if a unit of length (1'') represents a unit of force (lbs.).

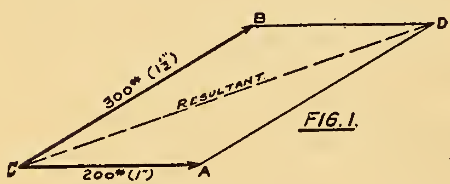


FIG. I.

Take two forces A (200 lbs.) and B (300 lbs.) acting on the same point with an angle between them. Let 1'' represent 200 lbs. of force, an arrow point the direction and "C" point of application of the forces. Then $AC=$ 200 lbs. or 1 inch, $BC=300$ lbs. or $1\frac{1}{2}$ inches. From C draw $BC=1\frac{1}{2}$ inches parallel to force B and draw $AC=1$ inch parallel to force A.

From A and B draw AD parallel to BC and BD parallel to AC, and from C draw the diagonal CD to the point of intersection; then CD is the resultant in direction and amount, and its length in inches multiplied by 200 lbs. (unit of length) gives the resultant force in intensity and direction.

If more than two forces in the same plane act on one point, proceed as before, taking two of the forces and finding the resultant; then combine this resultant with the next force and again find the resultant; continue in this manner until all the forces have been taken. The last resultant found is the resultant of all the forces.

An easier method is to arrange the forces (see Fig. 2) about the point of application, and proceed as follows: From the end of force No. 1 draw *2a*, parallel to *2*; from end of *2a* draw *3a*, parallel to *3*; then from end of *3a* draw resultant *R* to point of application *0*. To hold the forces in equilibrium, a force applied at *0* in opposite direction to the resultant *R* and of equal intensity is necessary. If more than three forces, use the same method until all have been considered.

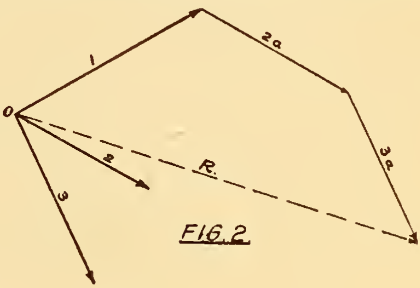


FIG. 2

INERTIA

The inability of a body to change its state of rest or motion.

WORK

The result of a force acting upon a body and producing motion is called work. The unit of work is the foot-pound, or the force of one pound moving a body through one foot of space.

Work (ft.-lbs.) = lbs. weight × ft. lifted.

Example. Thus 1 lb. weight lifted 10 ft. = 10 ft.-lbs. work
Also 10 lbs. weight lifted 1 ft. = 10 ft.-lbs. work
or 5 lbs. weight lifted 2 ft. = 10 ft.-lbs. work

For the purpose of comparing two operations, the work done may have been the same, yet the time for doing them may have been different.

Example. 100 ft.-lbs. work done in 1 minute as against
100 ft-lbs. work done in 5 minutes

Here the work done was equal, but the one took five times as long as the other. It will be seen that the rate of doing the work must be considered. This leads to the term power, which takes the time into account.

POWER

The product of the force (lbs.) multiplied by the distance (feet) multiplied by time (minutes consumed equals the power.

Expressed. P = Force × Distance × Time.

The unit of power used is the horse power (H. P.). This unit represents 33,000 foot-pounds of work done in one minute of time, and was adopted by James Watt for comparing the work of his steam engines with the work that could be done by London draft horses.

Example. 1000 lbs. raised 33 ft. in 1 minute, or 33,000 ft.-lbs. per minute equals one horse power.

MECHANICAL POWERS

Are those elementary machines used by man to modify motions and change the magnitude of forces, as follows:

Lever—This is the most commonly used of the mechanical powers, and consists of a bar supported at some point called the fulcrum. Its use is to modify a motion or force by means of power applied; the power and weight respectively being applied at points distant from

the fulcrum. These distances are known as the power and weight arms, respectively, and must be measured at *right angles* to the direction in which the power and weight, respectively, act and are not necessarily the actual physical lengths of the arms. The wheel and axle and gear wheels embody the principle of the lever also.

The principle is applied in three ways, known as classes (see Fig. 3):

Class 1. In which the fulcrum is between the weight and the power, which move in opposite directions.

Class 2. In which the weight is applied between the power and the fulcrum. Here both weight and power move in the same direction.

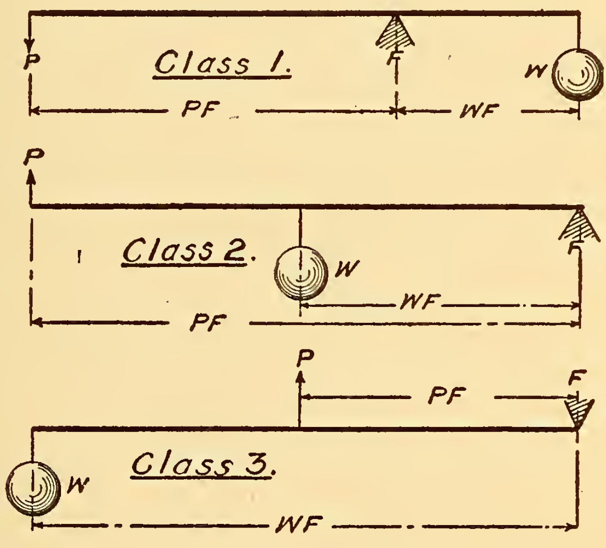


FIG. 3.

Class 3. In which the power is applied between the weight and the fulcrum. Here also the weight and power move in the same direction.

In Class 1 the power and weight may be equal or unequal, and either the one or the other may be the greater, depending upon the relative lengths of the two arms.

In Class 2 the power may be equal to or less than the weight, depending upon the lengths of the arms.

In Class 3 the power must equal or be greater than the weight.

The actual weight of the lever itself and friction are not considered in the study of the action of levers.

Law—One law governs all three classes and may be stated as follows:

Weight (W) multiplied by the Weight Arm (WF) equals the Power (P) multiplied by the Power Arm (PF). Where F represents the fulcrum. The formula is generally expressed,

$$W \times WF = P \times PF$$

If any three of the above are known, the fourth may be readily found.

Examples. Class 1. $W = 50$ lbs. $WF = 4$ ft. $PF = 8$ ft.

Then $50 \times 4 = P \times 8$ or

$P = \frac{200}{8}$ or 25 lbs.

Class 2. $W = 50$ lbs. $WF = 4$ ft. $PF = 8$ ft.

Then $50 \times 4 = P \times 8$ or $P = \frac{200}{8} = 25$ lbs.

Class 3. $W = 50$ lbs. $WF = 8$ ft. $PF = 4$ ft.

Then $50 \times 8 = P \times 4$ or $P = \frac{400}{4} = 100$ lbs.

Levers may be straight, bent or curved. The arms in any case must be considered as before stated, i. e., the straight distances from the fulcrum to points opposite the weight and power, respectively, and at right angles to their lines of direction. (See broken lines in Fig. 4.)

Pulley—The pulley is used where bodies (weights) are to be moved over longer distances, and consists of a wheel over which a rope or its equivalent is passed; to the one end of which the weight is attached and to the other end the power is applied.

Pulleys are arranged singly or in pairs or groups. The center or pin on which a pulley turns is called the *Axis*. The pulley may be known either as *Fixed;* if its axis does not move with the weight; or as *Movable*, if its axis does move with the weight.

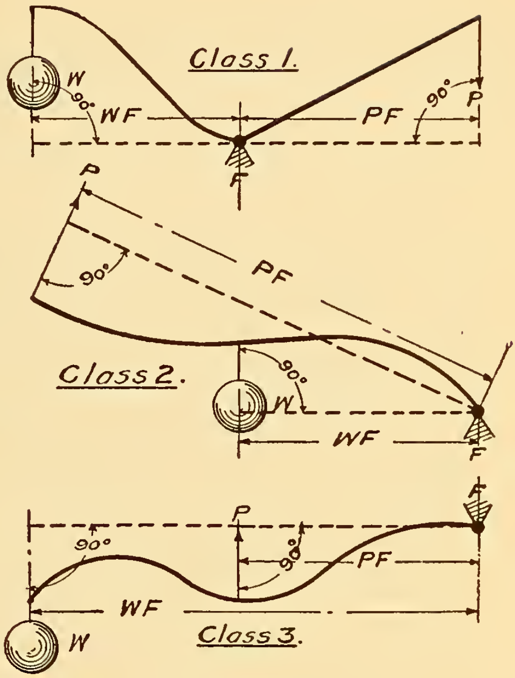


FIG. 4.

A *fixed* pulley changes the direction of the pull only, and has no mechanical advantage, because the weight and power move over the same distance.

A *movable* pulley has a mechanical advantage, because the weight moves only one-half as fast as the power or one-half the distance over which the power moves.

The fixed pulley is an example of a lever of the first class, with the weight and power arms equal.

The movable pulley is an example of levers of Class 2 or 3, i. e.:

If of Class 2, one end of the diameter of the pulley represents the fulcrum; the axis the point of application of the weight, and the opposite end of the diameter the point of application of the power. The power arm is, therefore, twice as long as the weight arm and the power will consequently move twice

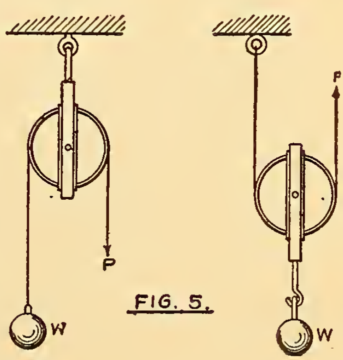


FIG. 5.

as far as the weight with one-half the effort (see Fig. 5). The same amount of work is done whether the weight is lifted direct or whether a fixed or movable pulley is used; as weight × distance lifted = power × distance through which it acts.

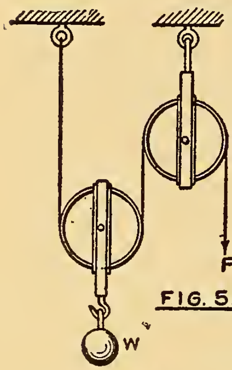


FIG. 5A.

The advantage derived from a fixed pulley is the change from a lift to a pull; the advantage from the movable pulley is that only one-half the effort but exerted over a longer distance is required. The arrangement shown in Fig. 5A has no advantage over the single movable pulley, and the fixed pulley is provided merely to change the direction of the pull.

Block and Tackle—A combination of ropes and pulleys (sheaves) grouped together, usually consisting of one or more pulleys in a fixed block and one or more pulleys in a movable block, the latter attached to the weight to be lifted and a rope, one end of which is attached to the fixed block and then passed over movable and fixed pulleys alternately until all have been served; the free end acts as the pull rope. The number of passes of the rope *moving* toward the fixed pulleys and *away* from the movable pulleys produce a mechanical advantage, as will be understood from Class 2 of levers. (See Fig. 6a.)

The advantage to be derived may be found from the formula in which *P* represents Power; *W*, Weight; and *R*, the No. of Ropes moving away from the movable block and toward the fixed block.

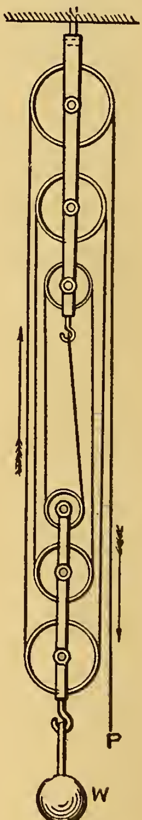


FIG. 6a.

Then Power = Weight divided by twice the number of ropes moving away from the movable block and

$$P = \frac{W}{2 \times R}$$

Example. A weight of 240 lbs. is to be lifted with a group of pulleys consisting of 3 fixed and 3 movable. What power is required on the pull-rope, and how many times the distance the weight moves must the end of the pull-rope traverse?

Solution. $P = \frac{240}{2 \times 3} = 40$ lbs. and pull-rope moves through six times the distance the weight moves.

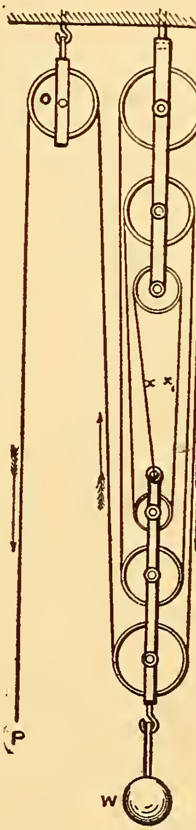


FIG. 6b.

The formula for any arrangement of pulleys is often given in the form

Power = Weight divided by the total number of ropes, less one. (It is to be understood that here, the last pass is supposed to have passed over a fixed pulley last.)

In the same example as before:

$$P = \frac{W}{\text{No. Ropes}-1} \text{ or } \frac{240}{7-1} = \frac{240}{6} = 40 \text{ lbs.}$$

In Fig. 6b. The first ropes (x) (x_1) and the arrangement do not satisfy Class 2, but do satisfy Class 1 of levers, and therefore the extra pulley "*0*" is required to produce the same direction of pull as in Fig. 6a. The end of the rope, therefore, would be

better placed if attached to the fixed block and not to the movable block as shown.

NOTE.—If the ropes passing over a movable pulley are not parallel and diverge from the pulley, some of the mechanical advantage is lost. When said divergence reaches 120 degrees – all the mechanical advantage is lost. The power (P) at any one instant may be found in two ways:

One by the formula: Power equals one-half the weight multiplied by the secant of the angle "a" or

$$P = \tfrac{1}{2} W \text{ (Sec. "}a\text{")} \quad \text{(See Fig. 7.)}$$

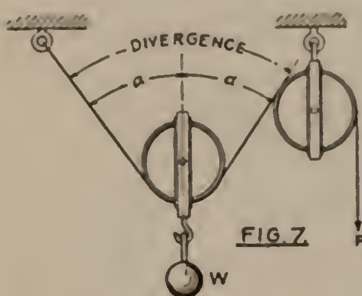


FIG. 7

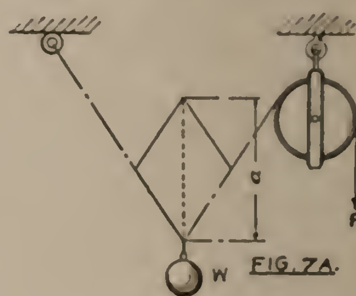


FIG. 7A.

The other by the use of the force diagram (see Fig. 7A), wherein the two cords leaving the movable pulley are represented by two of the sides of the parallelogram with which they are respectively parallel and the weight is represented by the length "a." This shows one of the many ways in which the force diagram may be used to solve various problems.

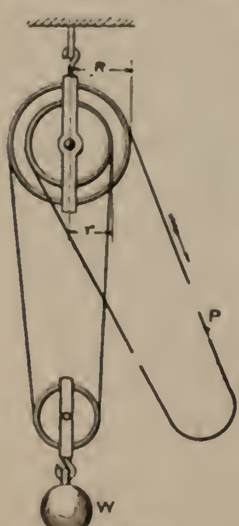


FIG. 8.

DIFFERENTIAL PULLEY BLOCKS

This machine (see Fig. 8) is used for lifting heavy weights. A chain must be substituted for the rope to avoid slip and in the fixed block two chain wheels of different radii R and r, one larger than the other, are used; both are on the same

spindle and fixed together so that one cannot turn without the other. The chain is continuous, starting from the movable pulley; thence over large fixed wheel to pulling section; thence (slack chain) to the small fixed wheel and back to the movable pulley.

The power $\times$ large radius R = one-half the weight multiplied by the difference between the large radius, R, and small radius, r, of the two fixed pulleys.

$$P \times R = \tfrac{1}{2} W \,(R-r) \text{ or } P = \frac{W(R-r)}{2R}$$

Example. $W = 240$ lbs. $R = 9''$ $r = 6''$

Then $P \times 9 = \dfrac{240\ (9-6)}{2}$ $P = \dfrac{240 \times 3}{18} = 40$ lbs.

WINDLASS

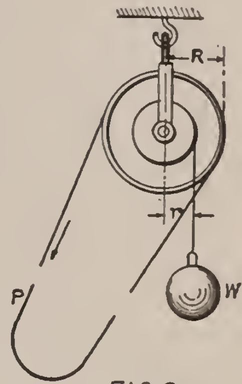


FIG. 9a.

A machine embodying the principle of levers under the term of wheel and axle. It has been and still is extensively used in place of pumps on farms to lift buckets of water from wells, also by the builder to lift his materials of construction. It consists of a drum or cylinder fixed on an axle and made to rotate by a pulley, crank arm or a series of lever arms also fixed on the axle. As the drum is made to turn, the lifting rope with one end fixed to the drum and the other end to the weight to be lifted, is gradually wound upon the drum, thus causing the weight to be lifted. The radius of the drum constitutes the weight arm, the axle provides the fulcrum and the radius of pulley or the length of the crank arm provides the power arm. If a gear (toothed wheel) is used

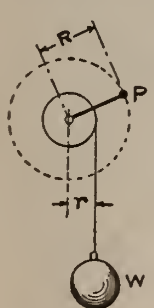


FIG. 9b.

in place of the pulley or crank, each tooth would represent a lever arm which would be successively brought into action. (See Fig. 9a–b.)

The same law that governs levers also applies here—

$$\text{Power} \times \text{Power Arm} = \text{Weight} \times \text{Weight Arm}$$

$$\text{or } P \times R = W \times r$$

$$\text{Also} \quad P = \frac{W \times r}{R} \quad \text{and} \quad W = \frac{P \times R}{r}$$

Example. Weight = 100 lbs. $R = 15''$ $r = 3$

$$\text{Then} \quad P = \frac{100 \times 3}{15} = 20 \text{ lbs.}$$

Now as the weight in pounds multiplied by the distance in feet it is raised = the ft.-lbs. of work performed; so also must the power in pounds multiplied by the distance in feet it acts = said work, and therefore the power and weight will be in the same proportion as the relative distances traversed. This holds equally true in the cases of levers, pulleys and wheel and axle; the mere fact that in the latter we have the circumferences of the drum and wheel to consider does not alter the relationship, as may be shown by the example as above: Weight 100 lbs., Radius of wheel 15'' (R) or 30'' diameter, Radius of drum 3'' (r) or 6'' diameter.

Now for one revolution we have:

$$\text{Work performed in ft-lbs.} = 100 \text{ lbs.} \times 6'' \div 12 \times 3.1416$$

$$\text{Also} \quad \text{Work performed in ft.-lbs.} = \text{Power(lbs.)} \times 30'' \div 12 \times 3.1416$$

$$\text{And} \quad \frac{100 \times 6 \times 3.1416}{12} = \frac{P \times 30 \times 3.1416}{12}.$$

$$P = \frac{100 \times 6 \times 3.1416 \div 12}{30 \times 3.1416 \div 12} = 20 \text{ lbs.}$$

Ratio of circumference of drum to circumference of wheel, as will be seen, is as 1 to 5 and power equals one-fifth the weight, or $100 \div 5 = 20$ lbs.

INCLINED PLANE

This one of the mechanical powers consists of an inclined surface to facilitate the lifting of an object or a weight from a low surface to one higher where it would be inconvenient or impossible to lift the weight vertically—as for example, the transfer of heavy boxes and barrels from a pavement to the platform of a wagon; the movement (lifting) of a car from the bottom to the top of a hill; the splitting of wood by an ax—the sides of the ax (wedge) are the surfaces of the inclined plane.

In the study of the inclined plane two methods may be used for finding the power necessary to hold in position a weight thereon, and also the intensity with which the weight tends to break through the surface. It will be seen readily that if the plane is nearly vertical the tendency to break through it would be small, but the lift would be more direct and, too, the plane would be short but steep, and the power would nearly equal the weight. On the other hand, if, with the same height the plane be long and therefore nearer horizontal, the reverse of the above would be true.

The two methods for finding power and pressure are: one the application of the parallelogram of forces and the other by the application of trigonometry, or the relation of the sides of the triangle, i. e., the length of inclined surface, the vertical height and the horizontal distance.

If, now, we take the diagram (Fig. 10), *A-B* is the incline; *A-C*, the horizontal distance; *B-C*, the height; and *W*, the weight, concentrated at its central point or cen-

ter of gravity. The weight W in lbs. may be represented by the length of a vertical line W-E as the resultant of the parallelogram; the two sides of such parallelogram Wx and Wy may be found readily, the former being parallel to the plane and the latter at right angles thereto. Now, by taking the same unit of length per lb. or per 100 lbs., which indicated the weight; Wx and Wy will represent the intensity with which the weight tends to slide down the incline and the tendency to break through the plane at right angles to the surface. The intensities represented by Wx and Wy would equal the power required in each case but acting in the opposite direction.

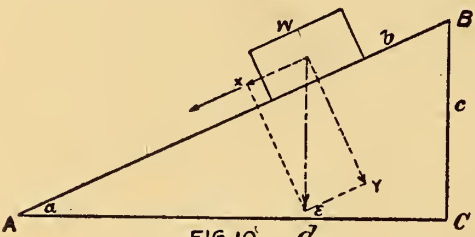


FIG. 10.

The Second Method—The power P required to prevent the weight W from sliding down the incline is weight multiplied by height in feet, B-C, divided by the length in feet, A-B, of the incline, or

$$P = \frac{W \times c}{b} \quad \text{also} \quad P_1 = \frac{W \times d}{b}$$

where P = the tendency to slide down the incline in pounds

P_1 = the tendency to break through

W = Weight in pounds

b = the length A-B

c = the height B-C

d = the length A-C

c and d, respectively, represent the sine and cosine of the angle "a"; also sides b, c and d bear the same relation-

ship to each other as the sides and diagonal *WE*, *Wx* and *Wy* of the parallelogram of forces.

NOTE.—The force *P* as found would in practice be more than necessary to prevent the weight sliding down the incline, but less than actually required to move the weight up the incline, because, first, we have merely provided for balancing the force exerted down the incline, and, second, we must provide also sufficient extra power to overcome the friction between the weight and the inclined plane, which retards movement of one object upon another and differs with the kinds of material in contact, with the nature of the surfaces in contact and the pressure on the surfaces.

FRICTION

The resistance to the movement of one body upon another. As a force is required to overcome it, it is often referred to as a force. The force required to overcome friction varies with the pressure, the nature of the surfaces in contact and the materials.

Sliding Friction—If two blocks of wood, each of same size and weight, one rough and the other smooth, be placed and made to slide on a rough board, a greater effort is required to keep the rough block moving than is required for the smooth one. If the smooth block be made to slide on the same board, but the surface thereof be made smooth, even less effort is required. This could be further reduced if a lubricant be introduced between the surfaces. Friction is also less if the materials are of different natures, also if the surfaces be hard and unyielding.

As the surfaces and nature of the materials in any one case may vary somewhat, the actual force required may vary also. In order to compute the force to overcome friction certain co-efficients are used, represented by the letter "*f*." These coefficients are given as a per cent. or so many 1–100 parts (expressed by decimals) of the pressure

between the surfaces for the varying conditions met with in practice. These coefficients "f" were deduced through experiment by using a plane surface or board, placing the weight thereon and then raising one end of the plane until the weight begins to slide; the angle of inclination of the plane to the horizontal is called the angle of repose, designated by "a" (see Fig. 10 also).

The tangent of this angle equals the coefficient of friction "f."

This is expressed by the formula:

$$f = \frac{c}{d}, \text{ but } \frac{c}{d} = \frac{Ey}{Wy} \text{ or } \frac{Wx}{Wy}$$

but Wx represents the force along the plane, and Wy the weight normal to the plane. The actual weight of test block is represented by the line WE, and in compliance with the laws of the inclined plane, the weight normal to the plane is Wy and equal to the cosine of angle "a" times the weight, and the force Wx along the plane in like manner is equal to sine "a" times the weight.

Now, if the normal pressure Wn of a body sliding on a surface multiplied by the coefficient of friction for any condition of material and surface is found, this will represent the power P_f required to overcome the friction alone. It may be expressed by the formula:

$$P_f = fW_n$$

A few coefficients (f) taken from "Machinery Handbook" give:

Bronze on bronze	0.20
Bronze on cast iron	0.21
Cast iron slightly lubricated	0.15
Hardwood on hardwood dry	0.48
Leather on hardwood dry	0.33
Leather on cast iron	0.56

Other coefficients may readily be obtained from any of the recognized handbooks.

NOTE.—The above conditions hold good whether the surfaces be flat and slide one upon the other or whether the surfaces are curved as in journals. The friction of rest is somewhat greater than the friction of motion—which is readily perceived in starting a sliding piece from a state of rest.

The laws of friction:

1. Under moderate pressures the friction is proportional to the pressure—with excessive pressures the coefficient of friction rises rapidly.

2. The total friction is independent of the area of the surface under moderate pressures.

3. Under low velocities the friction is not dependent upon the velocity, while under high velocities the friction decreases.

ROLLING FRICTION

When a wheel rolls along a surface, the perimeter of the wheel and surface are each somewhat depressed at the point of contact, which necessitates what may be termed a constant climbing due to this depression of the surfaces. It will be readily seen (see Fig. 11A) that the greater the diameter of the wheel the less will be the force required for the climbing action due to the increased length of the lever arm, as represented by the radius of each of the two circles representing two wheels with the power applied at the axle in each case. In like manner, the same holds true when the power is applied at the top of the wheel (see Fig. 11B).

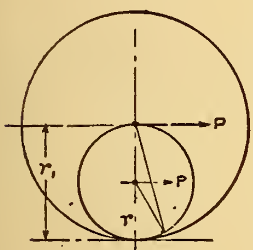


FIG. 11A.

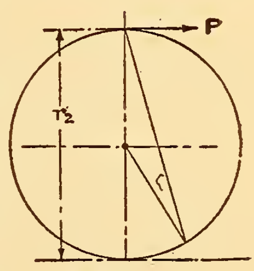


FIG. 11 B.

When the power is applied to the axle, sliding friction must also be considered, as this forms a bearing. Where the power is applied to the rim, rolling friction for the upper surface must be taken into account.

THE SCREW

This machine in practice really combines two of the mechanical powers, viz: the inclined plane and the lever, as in the Screw Jack, Fig. 12c. If an inclined plane (see Fig. 12a) be wrapped around a cylinder, the base of the

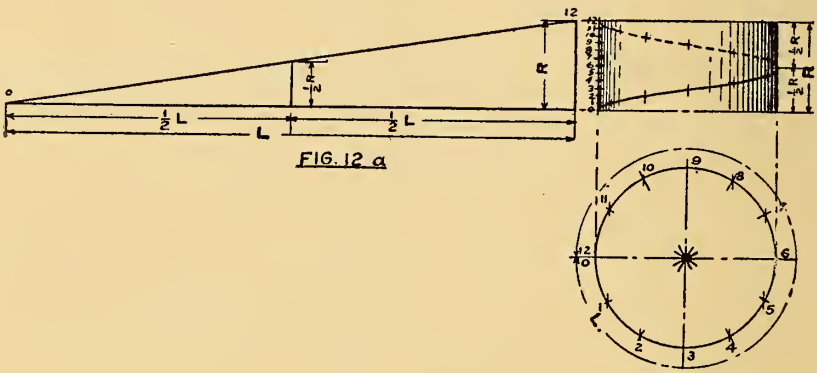


FIG. 12 a

FIG. 12 b.

plane—equal to the circumference of the cylinder and the height of the plane equal to the forward movement of the screw in one revolution, or lead, its upper edge will form the helix (see Fig. 12b). In practice the inclined plane is arranged as a screw thread, in that the screw may accommodate a nut with similar threads on the interior.

If a weight is supported by the screw and lifted thereby, during one revolution the screw will advance the distance represented by the lead and will lift the weight through a like distance. Now, if the lead were 1 inch for each revolution, and the weight were 2,400 lbs.

—the work done would equal the weight lifted in pounds times the distance moved in feet, or:

$$P = 2{,}400 \text{ lbs.} \times 1'' \div 12 = 200 \text{ ft.-lbs. of work}$$

Power in ft.-lbs. (P) = weight $(W) \times$ distance in feet (D) neglecting friction.

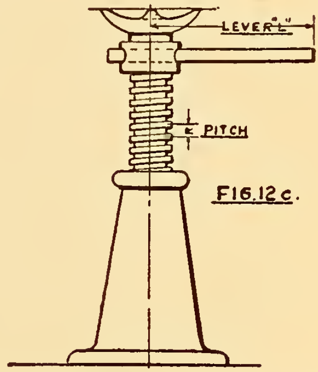


FIG. 12c.

Now, as a wrench or lever L (Fig. 12c) is necessary to turn either the screw or its engaging nut, the length of such wrench or lever in feet multiplied $\times\ 2 \times 3.1416$ would give the distance the lever arm has moved to do the 200 ft.-lbs. of work in one revolution. Now, if the lever were 2 feet long, the distance traversed would equal $2 \times 2 \times 3.1416 = 12.5664$ ft. and the power (P_L) required at the end of the lever would be:

$$P_L = \frac{P}{L \times 2 \times 3.1416} \quad \text{or}$$

$$P_L = \frac{200}{12.5664} = 16 \text{ lbs. about}$$

But on account of the friction between the threads the power (P_L) would have to be increased. Also the friction either at the end of the screw or the bottom surface of the nut would have to be considered, depending upon which one is turned.

For purposes of construction and strength, screws may have one or more threads and be known as single, double or treble, etc., threaded. In any case, the distance in feet traversed by the screw in one revolution must be considered and this is determined by any one thread.

In the problem given, the weight for one revolution of the screw was lifted 1 inch or 2,400 inch pounds, but as

work and power are usually in foot-pounds, the inch-pounds (2,400) must be divided by 12 (inches per foot) to obtain foot-pounds of work done. If inch-pounds are considered, the length of the lever arm L in the second part of the problem should be expressed in inches in place of feet, i. e., 24 inches.

By combining the two parts of the formula

$$P_L = \frac{W \times D}{L \times 2 \times 3.1416} = \frac{2400 \times 1''}{24 \times 2 \times 3.1416} = \frac{50 \times 1}{3.1416} = \begin{matrix}\text{16 lbs.}\\ \text{nearly}\end{matrix}$$

D and L being expressed in the same unit of length.

If in any case, the power at the end of a lever; the length of the lever and the lead of the screw are known; the weight which may be lifted, neglecting friction, may be found by rearranging the formula and using it in the form:

$$W = \frac{P \times L \times 2 \times 3.1416}{D}$$

Example 1—What weight can be lifted by a screw if a pull of 8 lbs. (P) is exerted at the end of a lever arm 24'' long (L) which turns a single thread screw having 2 threads per inch or a lead of $\frac{1}{2}''$ (D) per revolution?

$$W = \frac{8 \times 24 \times 2 \times 3.1416}{\frac{1}{2}''} = 8 \times 24 \times 2 \times 3.1416 \times 2 = 2412.74$$

NOTE.—The result would have been the same had the power been 16 lbs. and the lever arm 12 inches, or had the power been 16 lbs. and the lead 1'' (1 thread per inch).

MOTION

A body moving or changing its relation to another body is said to be in motion. A ball thrown from the hand, an automobile running along the road, or a wheel turning on its axis, are each in motion relative to some

other object; such as the earth or objects thereon or as the wheel relative to the axle.

No body will change its position and motion result unless acted upon by some force. A force always tends to produce motion or to change the direction thereof.

Laws of Motion—To Sir Isaac Newton, an English scientist, we are indebted for three laws, as follows:

First—Every body continues at rest or in uniform motion in a straight line unless acted upon by some force which may tend to change said state of rest or uniform motion. "*a*"

Second—A change in motion and the direction of said change is proportional to and in the direction of the force producing the change "*b*".

Third—To every action there must be an equal reaction and from the opposite direction "*c*."

"*a*"—The ball, the automobile or the wheel has no power to start moving, or if moving, to move faster, or slower, or to stop unless acted upon by some force. Weight (gravity) and a push or pull tend to produce motion or increase it. Friction and work tend to retard or to destroy it.

Increasing the motion is called "Acceleration." Decreasing the motion is called "Retardation."

"*b*"—A moving object, if acted upon by a force at an angle to the direction of its movement, will be deflected from its course in proportion to the intensity of the second force. This can readily be seen in the resolution of forces.

"*c*"—A ball held in the hand is pushed upward with the same force as its weight, due to gravity, tends to push the hand down.

The fulcrum of a lever pushes in the opposite direction and with the same intensity as the forces which tend to move it. If this were not true, in each case movement of the fulcrum would result due to unbalanced conditions.

As time entered into consideration with "work" and "power (HP)," so also must it be taken into consideration when dealing with motion—that is, the amount of motion or distance covered in a unit of time or its rate must be considered. This rate of motion is called velocity.

VELOCITY

If two trains both alike go in the same direction and for the same distance on parallel tracks with no stops, and the one takes less time than the other, its rate of motion or its velocity would consequently be greater than the slower train.

In considering the speed or velocity of trains, the distance is usually expressed in miles and the time in hours; and the rate of speed in "miles per hour."

In steam engines, it is the travel of the piston in "feet per minute."

In mechanics, velocity is expressed in "feet per second" for the reason that in this form we can compare it with the velocity resulting from a like object falling vertically due to the force of gravity.

If a train runs 60 miles in 2 hours, its rate would be:

$$V(\text{miles per hours}) = \frac{60}{2} \text{ or 30 miles per hour.}$$

The rate in feet per second is usually expressed by the *plain* letter "V."

Now, in one hour there are 3600 seconds, and in 30 miles there are $30 \times 5{,}280$ ft. or 158400 ft.

Then $3600\ V = 158400$, and $V = \dfrac{158{,}400}{3600}$ or 44ft. per sec.

Again, if an object moves over a distance of "h" feet in "t" seconds, whether circling around a central point or whether in a straight line, the velocity in feet per second may be expressed:

$$V(\text{vel. ft. per second}) = \frac{\text{feet } (h)}{\text{Sec. } (t)}$$

$$\text{or } V = \frac{h}{t}$$

Example—A body travels 100 ft. in 5 seconds; therefore $V = \frac{100}{5}$ or 20 feet per second.

The same formula may be expressed $t = \frac{h}{V}$ or $h = t \times V$, depending upon which factor is required. It is understood two of the factors must be known.

Take the above example. If the velocity and distance passed over had been known, the time $t = \frac{100}{20}$ or 5 seconds. In like manner, if the velocity and time had been given to find the distance gone over, then the distance $h = 5 \times 20$ or 100 ft.

In the above examples, both the train and the object, when moving at the velocities as found, i. e., 44 ft. and 20 ft. per second, relatively, would, according to Newton's first law of motion, continue at this rate forever, provided no external force act thereon either to increase (accelerate) or decrease (retard) their motion. Nor should any power be required to keep up this rate. The above is not obtainable, as the resistance of the air, the sliding and rolling frictions of the journals, and track, all help to retard the movement of the train. In a like manner, the friction between the parts of a steam engine, line shafting, machine and the final work to be

done, all call for the expenditure of energy; so that, to keep the train or the engine moving at the uniform speed, it requires the constant application of sufficient power to overcome such resistance or load, otherwise the train or engine would slow down and finally stop. On the other hand, if part of the load were relieved without an equal reduction in the power applied, the speed or velocity would increase or accelerate until the parts, especially the flywheel of a stationary engine, would fly apart with disastrous results. To provide against the slowing down on the the one hand and against excessive speed on the other, engine governors are used. The governor acts also to conserve the energy stored in the steam. If the excess of power above that required remain uniform, the velocity would increase at a uniform rate.

VELOCITY OF FALLING BODIES

Gravity, as before stated, is the force of attraction between all bodies and the earth, and causes them to have weight. This force, too, naturally causes an acceleration in the speed or velocity of the falling body, and at a uniform rate of 32.16 feet for each second of time. If a body starts falling from a state of rest, its velocity at the start would be *0* feet per second; at the end of the 1st second its velocity would be 32 feet per second (in round numbers); at the end of the 2nd second it would be 64 feet per second; at the end of the third, 96 feet per second, and so on.

If the velocity at the beginning of the 1st second is 0 ft., and at the end of the 1st second is 32 feet, the space passed through during 1st second is 16 ft. or $(0+32)\div 2=$ 16 ft. per second—this being the mean velocity for the 1st second.

The mean velocity for the 2nd second is $(32+64)\div 2=$ 48 ft. per second; and mean velocity for 3rd second is $(64+96)\div 2=80$ ft. per second, and so on. Each second the velocity increases 32.16 feet per second.

It will also be seen that at the end of any number of seconds of time the velocity would equal the number of seconds multiplied by the rate of acceleration, 32 feet (32.16' actually) per second—so if V represents the velocity in feet per second; t, the time; g, the effect of gravity, then $V=g\,t$, and the velocity at the end of the 3rd second would be

$V=3\times 32$ or 96 ft. and the mean velocity designated V_m for the full period would equal $V_m=\frac{t\,g}{2}$ or $\frac{96}{2}=48$ ft.

and if h represents height of fall in feet, then

$$h=V_m\times t \text{ or for 3 seconds } h=48\times 3 \text{ or } 144 \text{ ft.}$$

but V_m (as before) $=\frac{t\,g}{2}$, so that the formula may be expressed

$$h=\frac{t\,g}{2}\times t \text{ or } \frac{g\,t^2}{2} \text{ or } \tfrac{1}{2}\,g\,t^2$$

As $V=g\,t$ and $t=\frac{V}{g}$, by substitution in the preceding equation

$$h=\frac{g\left(\frac{V^2}{g^2}\right)}{2}=\frac{\frac{g}{1}\,\frac{V^2}{g^2}}{2}=\frac{\frac{V^2}{g}}{2} \text{ and } h=\frac{V^2}{2g} \text{ and}$$

$$2gh=V^2 \text{ or } V=\sqrt{2gh}$$

Now, as $g=32.16$, then $2g=64.32$ $\quad V=\sqrt{64.32\times h}$, but the square root of 64.32 is 8.02, so the formula is often used in the form

$$V=8.02\sqrt{h}$$

Now, if the distance "h" through which a body is falling or moving is known, its velocity by the last formula may readily be found—as, for instance, it was ascertained that in three seconds of time the distance h was found to be 48×3 or 144 ft.

48 ft. being the mean velocity and 3 the time in seconds,

$V = 8.02\sqrt{h}$ or $V = 8.02\sqrt{144}$ or 8.02×12 or 96.24 ft. per second, or a mean velocity (V_m) of $\frac{96.24}{2} = 48.12$ ft.

Should the time "t" be required, the above formulas may be written

$$t = \frac{V}{g} = \sqrt{\frac{2h}{g}} = \frac{\sqrt{h}}{4.01}$$

From the above it will be seen that a moving body, whose weight and velocity are known, may be compared to a falling body of the same weight falling through a distance of h feet, and that it can do the same amount of work as such body falling the same height.

All bodies would fall equally fast in a vacuum, but falling in the air their movement is retarded—those of greater density falling faster, due to their volume being relatively smaller. A lead ball and a feather will fall equally fast in a vacuum, but in air the lead ball will fall much faster, as the air would offer less resistance to its passage.

ENERGY

If a body of any weight "W" is lifted through "h" feet, the amount of work in ft.-lbs. expended would be $E = W \times h$. If the weight were allowed to fall, it would do an amount of work equal to that required when raising it; therefore the energy is said to be stored.

Example—If a weight of 200 lbs. were raised 50 feet, then $E = 200 \times 50 = 10{,}000$ ft.-lbs. of work or stored energy.

If the weight were allowed to fall, it would do equally as much work, or $200 \times 50 = 10{,}000$ lbs. In raising the weight no account of the time was taken, but when allowed to fall gravity determines the velocity in feet per second for any period of the fall, air friction not being considered.

If in the formula $E = Wh$, the equivalent of h, or $\frac{V^2}{2g}$, as found under Vel. of Falling Bodies, is substituted, the formula reads

$$E = W\frac{V^2}{2g}, \text{ but } g = 32.16' \text{ and the formula becomes}$$

$$E = \frac{WV^2}{64.32}$$

Example—If a body weighing 200 lbs. is falling or moving at a velocity 50 feet per second, and its motion be then arrested in one second of time, the power or energy expended would be at the rate:

$$E = \frac{200 \times 50^2}{64.32} = 7773.6 \text{ ft.-lbs. work per second}$$

Now 1 Horse Power $= 33{,}000$ ft.-lbs. per minute, or 550 ft.-lbs. per second, and $7773.6 \div 550 = 14.13$ *HP*, the rate for that one second of time.

This formula may also be applied to parts of machines moving in any direction, if the weight and velocity of the moving parts are known. The principle is observed in punching presses, drop hammers, in the shocks incident to collisions or in the sudden stopping or starting of machines.

In the foregoing attention was given to the storing up of energy and the power developed when the entire motion was arrested during one second of time, and as found in trip hammers and the like. In most machines, however, only that portion of the stored energy and motion is arrested as is sufficient to do the work required at any one period. This allows a continuance of movement in the source of the power and a consequent restoring of the energy used, preparatory to it again being called on to do work. It will now be seen why steam and gas engines, punching and printing presses, etc., are provided with flywheels, wherein the energy derived from the engine piston or from the driving belt is stored and is ready for instant use. Flywheels or balance wheels are made relatively large in diameter and with a heavy rim, in order to prevent any considerable reduction in speed when power is taken therefrom and also to prevent a sudden excessive increase in speed after the work is done. The rim represents a succession of weights on lever arms (spokes)—equal to a radius extending from the center of the shaft to the center of the rim. From the theory of levers, the greater the radius and the heavier the rim the less should be the disturbance for any change due to energy taken out or put in. The energy stored in flywheels, where E = total energy of flywheel in foot-pounds, W = weight of rim in pounds (spokes and hubs usually neglected), V = velocity of the rim in feet per second, (using the mean radius of rim to center of shaft), $g = 32.16$, then

$$E = \frac{WV^2}{2g} \text{ or } \frac{WV^2}{64.32},$$ or the total energy available if

the wheel is brought to a full stop, but flywheels usually are only partly reduced in speed and the energy taken

out would be equal to the difference between the total energy due to the full speed minus the energy left at the reduced speed. If now the velocity at full speed is represented by V_1, and the velocity at the reduced speed by V_2 and the energy taken as E_t, then

$$E_t = \frac{W(V_1^2 - V_2^2)}{64.32}$$

The governing of engines in their power output is determined by the speed changes in the revolutions of the flywheel—that is, if the speed decreases, more power is applied to the piston, and, *vice versa*, less power is applied with an increase of speed, below or above a fixed point called the rated speed and on which the horse power of engine is based.

The diameter (Radius×2) of a flywheel is limited mainly by its speed, for as the speed increases another force begins to act and, if excessive, destruction of the wheel would result. This force is known as centrifugal force.

CENTRIFUGAL FORCE

That force acting in a body rotating about a center tending to cause it to move in a straight line but prevented from so doing by the tie which connects it with the center of rotation or with parts adjacent to it. If the speed be great enough, the forces in the body which hold its particles together will be overbalanced and the body will fly to pieces, as in the "bursting" of flywheels, and as different materials vary in strength, their speed limits, too, would vary. The material most used is cast iron.

The action of centrifugal force may readily be observed by taking a weight or ball suspended from a string (see Figs. 13 and 14), the end of the string being held in the

hand. If the ball W be started to swing around in a horizontal circle, the faster the speed becomes the higher will be the plane of rotation until such speed is reached when it will swing in a horizontal circle level with the

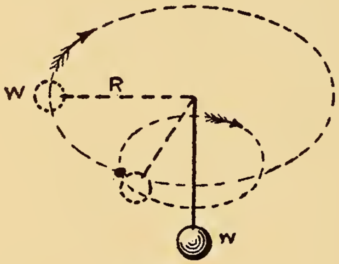


FIG. 13.

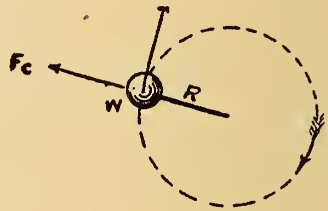


FIG. 14.

hand or point of support. Two things will be evident: 1st, through the speed the attraction of gravity has been overcome, and 2nd, the ball is pulling hard against the string and the strength thereof prevents the ball from flying into space and in a straight line tangent to the circle of rotation. This agrees with laws of motion as given before.

Centrifugal force may be expressed by the formula:

$F=\frac{WV^2}{gR}$ where F = the centrifugal force in pounds, W, weight of the revolving body

V = Velocity of body in feet per second around the axis

g = Gravity (32.16) as before

R = Radius in feet.

The formula may be written in the more convenient form:

$F = .000341WRn^2$ in which n = number of revolutions per minute.

Example—Flywheel rim weighs 500 lbs., revolutions per minute 120, radius 18'' ($1\frac{1}{2}$ ft.). Find centrifugal force.

$F = .000341 \times 500 \times 1.5 \times 120 \times 120 = 3682$ lbs. centrifugal force.

The centrifugal force increases with an increase of the velocity, and such increase in velocity may be due either to an increase in the number of revolutions or to an increase in the radius, at same rev. per min.

In the design of flywheels, a rim velocity of 85 feet per second is usually taken as a maximum, as with speeds much in excess of this the wheel may burst, due to the action of centrifugal force, as such force would exceed the tensile strength of the cast iron of which flywheels are usually made.

Any additional metal placed in the rim would also be stressed beyond its capacity due to centrifugal force.

In a flywheel rim, the total centrifugal force results from weight of entire rim, its velocity and radius. The force "D," tending to tear the wheel in two, would

$$= \frac{0.000341\,WRn^2}{3.1416}$$

As relationship between the diameter and circumference of a circle $= 3.1416$. Then $D = 0.00010854\,WRn^2$.

CHAPTER II

POWER TRANSMISSION

BELTS AND PULLEYS

These are extensively used to transmit power from one revolving shaft to another, or to a machine for doing work. Most machines and tools in shops and factories are driven by them (see Fig. 15–a–b–c–d).

FIG. 15a.

In the simplest form (Fig. 15a) the pulley, *d*, on one shaft is called the driver and that on the other the driven pulley, d_1. The pulley, *d*, is fixed on the shaft and is rotated thereby. Connecting this pulley with the driven pulley is a flexible belt, *e*, of leather, or other similar material. This belt when in position hugs the surfaces of the pulleys, and its ends being joined together, makes the belt endless.

Now, if power is applied to shaft of pulley, *d*, and it makes one revolution, then, due to the friction between the belt and pulley, any point on the belt will be moved through a distance equal to the circumference of the driving pulley, *d*, and in the same period of time. If the driven pulley, d_1, is of equal diameter, it will also make one revolution, except for any slip between the belt and the pulleys, as the circumference of the one pulley is the same as that of the other.

It will be readily seen, the principles of levers apply in each of the two pulleys. The center of the shaft in each case is the fulcrum.

If the driving pulley, d, is one-half the diameter of the driven pulley, d_1, the latter would make only one-half a revolution to one revolution of the driver, because its circumference is twice that of the driver, d.

If d = diameter of the driver; d_1 = diameter of driven; both in the same unit of measurement; n = number of revolutions of the driver in a given time and n_1 = number of revolutions of the driven in the same time and 3.1416, the relation between diameter and circumference;

Then in any case $3.1416 \times n \times d = 3.1416 \times n_1 \times d_1$

Example—Driver 12'' diameter; No. Rev. Driver 100 r.p.m. and driven 20'' diameter

Diameter: $3.1416 \times 100 \times 12'' = 3.1416 \times n_1 \times 20''$

$$3.1416 \quad (1200) = 3.1416\,(20 \times n_1)$$

$$62.832\ n_1 = 3769.9200$$

$$n_1 = \frac{3769.9200}{62.8320} = 60 \text{ r.p.m.}$$

Now, as the factor 3.1416 is common to both wheels, it may be eliminated and the formula be expressed in simpler and better form—

$$n_1 \times d_1 = n \times d \quad \text{or} \quad n_1 = \frac{dn}{d_1}$$

or in the same example as above—

$$n_1 = \frac{12 \times 100}{20} = 60 \text{ r.p.m.}$$

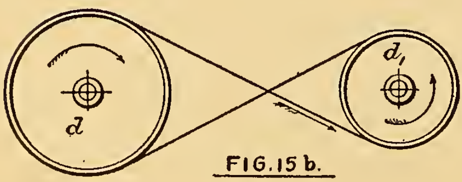


FIG. 15 b.

In case "*b*" (Fig. 15) the belt is crossed, instead of open as in Fig. 15a, and the pulleys turn in opposite directions instead of the same direction.

In the case (Fig. 15c) a common factory arrangement is shown. Here, it is only necessary to combine the processes and to express the formula as follows:

n(r.p.m.) of engine $\times d \times d_2 \times d_4 = n_1$(r.p.m. of mach.) $\times d_1 \times d_3 \times d_5$

In which d, d_2, d_4 are driving pulleys, and
d_1, d_3, d_5 are driven pulleys

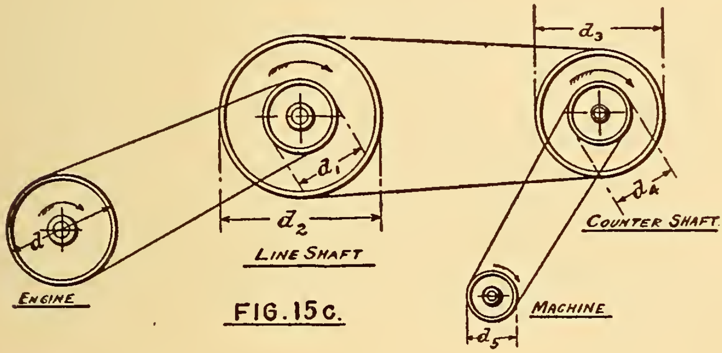


FIG. 15c.

This may be expressed—The product of the number of revolutions of first driver multiplied successively by the diameter of each successive driver must equal the number of revolutions per minute of the last driven pulley multiplied successively by the diameter of each driven pulley.

Example—Engine rev. per min. $n = 80$, driving pulley $d = 36''$
1st driven, $d_1 = 12''$, 2nd driver $d_2 = 16''$, 2nd driven, $d_3 = 12''$
3rd driver, $d_4 = 12''$, and 3rd driven, $d_5 = 10''$; to find the number r.p.m. n_1 of machine spindle?

Then by substituting in the formula $n \times d \times d_2 \times d_4 = n_1 \times d_1 \times d_3 \times d_5$; the r.p.m. and the diameters of the various pulleys, the condition would be

$$80 \times 36 \times 16 \times 12 = n_1 \times 12'' \times 12 \times 10$$

and $n_1 = \dfrac{80 \times 36 \times 16 \times 12}{12 \times 12 \times 10} = 384$ r.p.m. for the machine.

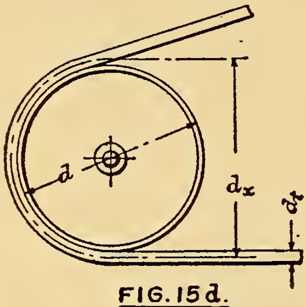


FIG. 15d.

If the r.p.m. n and n_1 are given and it is desired to provide a pulley train to satisfy the speed ratio, a number of combinations with different diameters may be used, provided the product for each of the two sides of the equation are equal.

NOTE.—In cases where the speed ratio between any pair of shafts is large and the one pulley is small and runs at high speed (see Fig. 15d), the diameter of the said pulley should be taken as the diameter d_x or $d+d_t$ (thickness of belt). This is found necessary frequently in electric motor drives—where the motor speed is high and the driving pulley small.

The power transmitted by belting is determined by the diameter of the pulley, the width of belt and thickness of belt and rev. per minute.

In belt drives, there is always some slip between the belts and the pulleys—this makes them impractical where a positive drive, as in timed mechanisms is essential. In cases of this kind, either a *sprocket chain* or a *gear* drive is used. In either case the pulleys are replaced by toothed wheels. In the former a chain is substituted for the belt, and in the latter no chain is used but the teeth are so shaped that those of one wheel mesh with those of the other wheel of the pair.

Should the drive shaft and driven shaft of a gear wheel train be too far apart, toothed idler wheels are frequently used in place of substituting a sprocket chain drive.

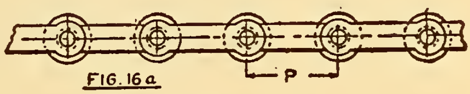
SPROCKET CHAINS AND WHEELS (PITCH OF)

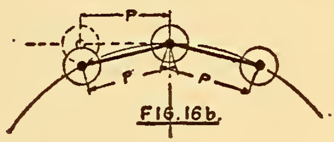
Belts of leather, etc., usually consist of one piece, with the two ends fastened together forming an endless band.

Sprocket chains consist of a series of links joined together to form an endless chain.

In Fig. 16—"*a*," a roller sprocket chain is shown, in which a roller is held on a cross bar or pin passing through the side links and held thereby.

P—denotes the pitch or center to center distance of the rollers.





The openings in the chain are for the reception of the teeth of the sprocket wheels on which the chain is to be used, and the pitch therefore determines the distance from center to center of such teeth.

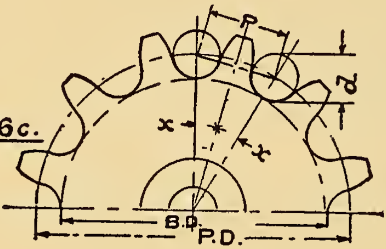
In *b*, Fig. 16—Two links of a roller chain are shown—the broken lines representing the links on a straight line and the full lines their relationship when passing over a wheel with the consequent shortening of the chain toward the inside and a corresponding lengthening on the outside of a circle whose circumference passes through the link center. This circle, called pitch circle, is the one on which the teeth of the wheel must be laid off and its circumference must be divisible by the pitch without a remainder, so that the number of teeth and number of spaces will be equal, and all teeth be alike. The teeth prevent slip between the chain and wheels; consequently, for any turning movement of the driver there will be a like turning movement of the driven wheel and the number of teeth (T) in the driving wheel multiplied by its number of revolutions (R) will equal the number of

teeth (T_1) in driven wheel, multiplied by its revolutions (R_1), or

$$T \times R = T_1 \times R_1 \text{ and } R_1 = \frac{T \times R}{T_1} \text{ or } T_1 = \frac{T \times R}{R_1}$$

The pitch circle in roller chain sprocket wheels lies half way between the top and bottom of the tooth. The diameter of the rim of the sprocket wheel (see Fig. 16c) at the bottom of the tooth is called the base diameter and is equal to the pitch diameter minus the diameter d, of the roller. The outside diameter equals pitch diameter plus the diameter d, of the roller. If n = number teeth, P = pitch of chain, $2X$ = angle due to chord (pitch) on the pitch circle,

FIG. 16c.



$$\text{then} \quad 2X = \frac{360°}{n} \text{ and } X = \frac{360}{2n} \text{ or } \frac{180}{n}$$

$$\text{and the pitch diameter of wheel} = \frac{P}{\sin X} \text{ or } \frac{P}{\sin \frac{180°}{n}}$$

GEAR WHEELS—PITCH

There are two kinds of pitch in gear wheels —circular pitch ($C.P.$) (see Fig. 17), the curve distance from center of one tooth to the center of the next tooth measured on the pitch circle, and diametral pitch ($D.P.$) which indicates the number of teeth on the wheel for each inch of the pitch diameter.

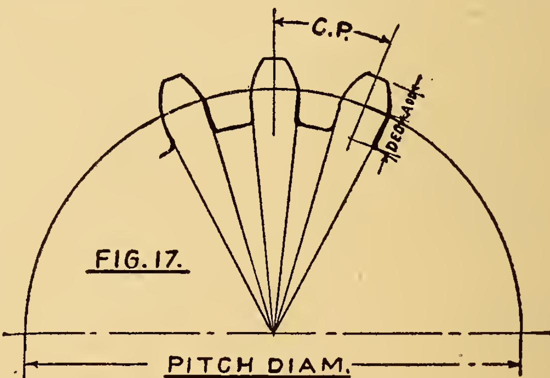


FIG. 17.

The pitch in gears, just as with sprocket wheels, determines the relative revolutions of the drive and driven wheels, and the same formula $T \times R = T_1 \times R_1$ holds good for gears also.

PITCH CIRCLE

The circle whose circumference may be divided by the circular pitch without a remainder, into as many parts as there are to be teeth in the wheel—such parts consisting of a tooth and a space.

The pitch circles of a pair of gears working together should be tangent to each other and the teeth of the two wheels be accurately formed so that the sides of those of one wheel may have a rolling contact with the sides of those of the other wheel.

PITCH DIAMETER

The diameter of the pitch circle may be found readily if the number of teeth, n, and either the circular pitch or the diametral pitch is given. If n = number of teeth and $C.P.$ = circular pitch, then $n \times C.P.$ = circumference of pitch circle, and circumference $\div$ 3.1416 = pitch diameter

or pitch diameter $= \dfrac{n \times C.P.}{3.1416}$, 3.1416 here refers to the relation between the diameter and circumference of any circle.

Again, if n = number of teeth and $D.P.$ = diametral pitch, then n divided by $D.P.$ = pitch diameter;

or, pitch diameter $= \dfrac{n}{D.P.}$

As both these formulas give the pitch diameter, they are equal to each other and a fixed relationship between

the circular pitch (*C.P.*) and diametral pitch (*D.P.*) may be found, so that if one is given the other may be found as follows:

$$\frac{n \times C.P.}{3.1416} = \frac{n}{D.P.} \text{ or } n \times C.P. \times D.P. = 3.1416 \times n$$

As n is common to both sides of the equation, it may be eliminated and $C.P. \times D.P. = 3.1416$ and

$$C.P. = \frac{3.1416}{D.P.} \text{ or } D.P. = \frac{3.1416}{C.P.}$$

If the gear teeth are on a straight bar and the pitch line consequently is also a straight line, the pitch circle is considered as of an infinite radius and the gear is called a rack. The pitch circle in gears lies between the top and bottom of the teeth; the part outside the circle is called the addendum or face, and the part inside of the pitch circle the dedendum or flank (see Fig. 17). This permits the teeth of the two wheels to mesh and the pitch circles to become tangent to each other, if the teeth and the spaces between them are properly formed, by permitting the face or ends of the teeth of one wheel to pass into the space between the teeth of the other wheel and for the proper distance, and *vice versa*. The dedendum or flank is made greater than the addendum or face to permit clearance between the ends of the teeth and the bottom of the space.

In some gears, depending upon the class of workmanship, especially in cast gears, which are not machined afterward, a side clearance also is allowed—that is, the space is made wider than the tooth.

PROPORTION FOR TEETH

The basis for proportioning the teeth is the circular pitch (*C.P.*). The addendum is equal to $C.P. \div 3.1416 =$

.3183 $C.P.$ and likewise the bottom clearance $= C.P. \div 20$, and the dedendum becomes $\frac{(C.P.)}{20} + \frac{(C.P.)}{3.1416}$ or

$$\frac{3.1416C.P. + 20C.P.}{20 \times 3.1416} = \frac{23.1416C.P.}{62.8320} = .3683\ C.P.$$

and the whole depth of tooth $= (.3183 + .3683)\ C.P. =$ .6866 $C.P.$

The outside diameter of gear blank =

Pitch diameter $+ (2 \times .3183 \times C.P.)$ or

Pitch diameter $+ .6366 \times C.P.$

The thickness of tooth for rough gears $= .48 \times C.P.$, and the width of the space $= .52 \times C.P.$ The sum of which equals $1 \times C.P.$

In first-class finished gears, the thickness of the tooth and space, each equal $.5 \times C.P.$ or $\frac{C.P.}{2}$

The center to center distance between shafts =

$$\frac{\text{Sum of the pitch diameters}}{2}$$

TOOTH CURVES

Two general systems are used for finding the proper curves for the shaping of the teeth—the Cycloidal and the Involute are such systems. The Cycloidal system (see Fig. 18) is based on finding a circle, called generating circle, which is alternately rolled on the inside and outside of the

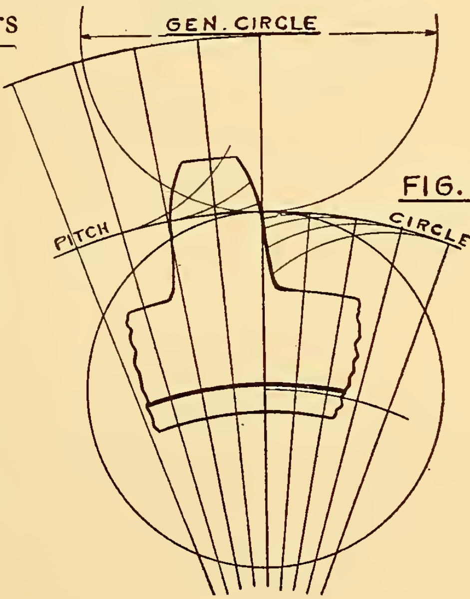


FIG. 18.

C 'CLOIDAL

pitch circles with a trace point, which describes two curves for each wheel, an Epicycloid Curve for the face, and a Hypocycloid Curve for the flank.

The diameter of the generating circle is found by taking one-half the diameter of a 12-tooth pinion and of the pitch being used (see Fig. 18). A generating circle based on the 12-tooth pinion generally is used, for with it the flanks of the teeth become radial.

THE INVOLUTE SYSTEM

In this system (see Fig. 19) a line drawn at an angle of $75\frac{1}{2}°$ to the line of gear centers defines the angle of approach of the teeth of the two wheels. This line crosses the line of centers at the point of tangency of the two pitch circles. A circle inside and concentric with the pitch circle drawn tangent to the line of contact is called the base circle. An involute curve starting from the point of intersection between the base circle and line of centers forms the curve for the addendum, or face, and part of the dedendum, or flank, of the gear tooth, respectively, of each wheel. The balance of the flank, or to the root of the tooth, is drawn radially toward the center of the gear. starting at the base circle.

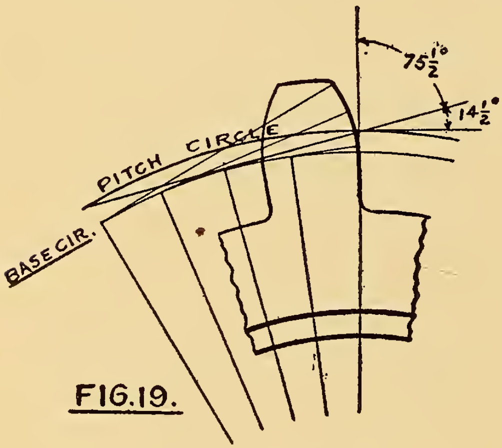


FIG. 19.

INVOLUTE.

Cycloidal teeth are considered weaker than involute, and for good operation the pitch circles must always be

tangent one to the other. Also 24 cutters of each pitch are required to cut gears ranging from 12 teeth to a rack.

Involute teeth are considered stronger and the pitch circles need not be exactly tangent one to the other for good operation. Again, only 8 cutters are required to cut gears ranging from 12 teeth to a rack, nor is side clearance usually allowed in cut gears, the tooth thickness being equal to one-half the pitch.

Involute teeth are used more extensively for the above reasons and practically universally so for bevel gearing. In the involute rack the sides of the teeth are straight and at right angles to the line of contact, except that the tops are often rounded off to prevent interference.

SPUR GEARS

In gear drive where two shafts are parallel and the teeth throughout their length are parallel to said shafts, the gears are known as Spur Gears.

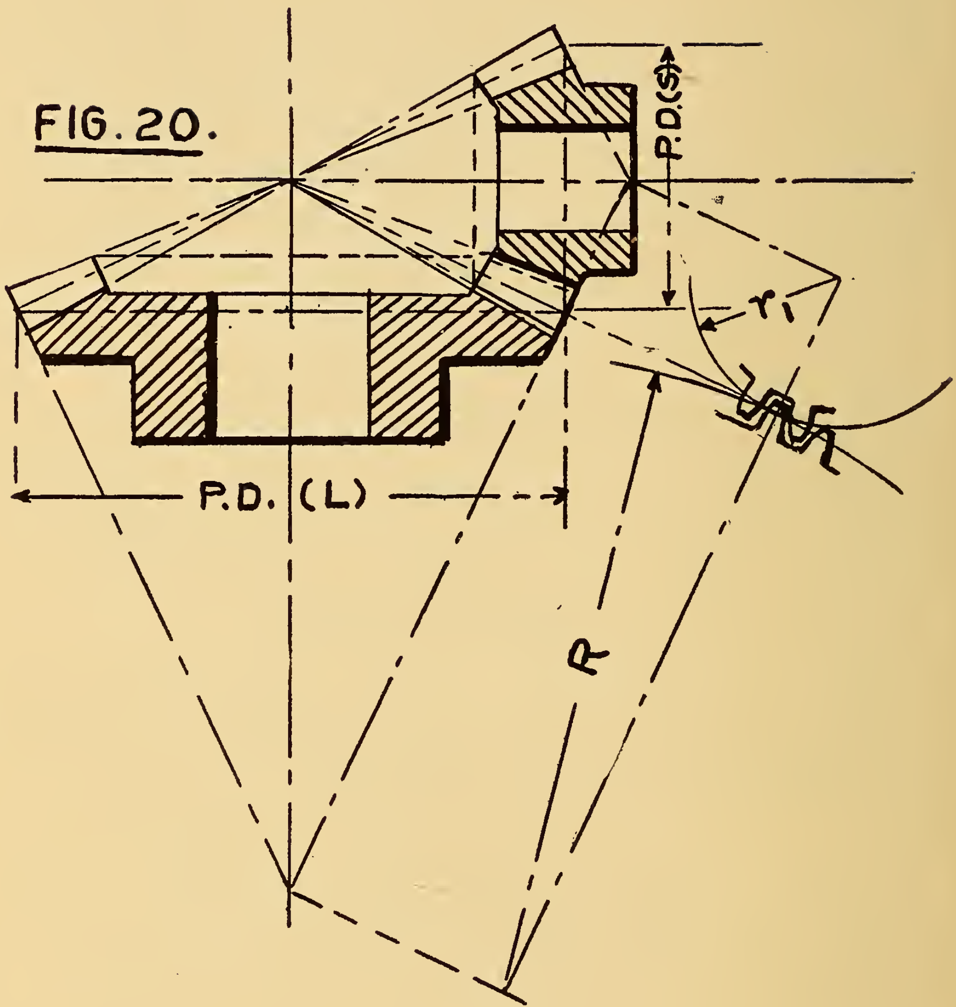
BEVEL AND MITRE GEARS

In drives where the shafts are not parallel and where their center lines, if projected, would intersect, are classed either as *bevel* or *mitre gears*, depending upon the angle of intersection and number of teeth in the mating gears. If the angle is 90° and both wheels have a like number of teeth, they are called *mitre gears*.

In spur gears the pitch circles, etc., are considered at the end of a short cylinder. In bevel and mitre gears they form the bases of the frustums of pitch cones whose apexes meet at the point of intersection of the two center lines (see Fig. 20).

In the illustration the pitch diameters *L* and *S*, respectively, form the bases of the pitch cones. The line of

FIG. 20.



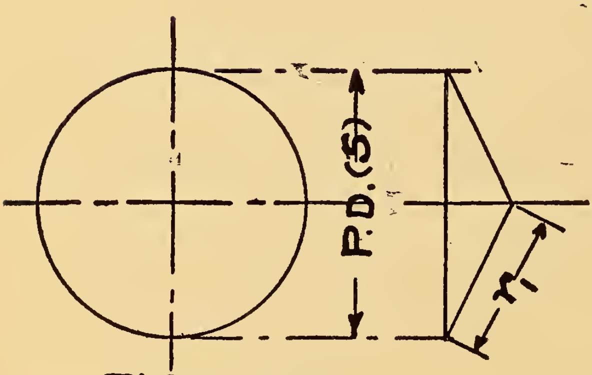


FIG. 20 B.

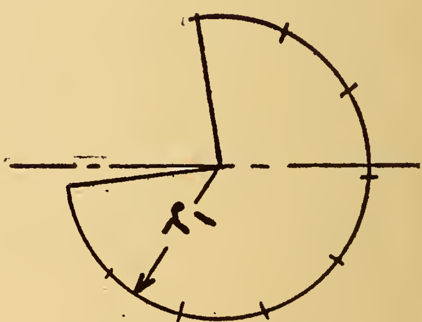


FIG. 20 C

tangency of the two cones is the pitch line, because at any point throughout its length it would define the relative diameters of the gear and pinion at that point.

The pitch diameters and number of teeth, as in spur gears, determine the speed ratio between the two shafts. The back or cutting face (see Fig. 20) of the gear and pinion, respectively, is at right angles to the pitch line and at the point of tangency of the two pitch diameter circles. On this face, above and below the pitch line, the addendum and dedendum of the teeth of each gear are laid off and in the same proportions as for spur gears. In like manner, the thickness of the teeth at cutting face would be the same as for spur gears. The teeth in each wheel would, however, decrease in size toward the point of intersection of the two shaft centers.

As before stated, the involute system is used almost universally for bevel gears. To develop the tooth curve, the pitch diameters L and S would serve well enough for the thickness of the teeth and width of space between them, but as they are at right angles to their respective shafts the addendum and dedendum for each wheel would be foreshortened, and the true tooth curve not be obtained. If, however, the development of the tooth curve be on a plane at right angles to the pitch line and at the point of tangency of the two pitch circles, said plane (cutting face) is common to both gears and the addendum, dedendum and thickness would show in their relative proportions. The cutting face of each, however, (see Fig. 20b) forms the side of a cone each having as its base the pitch diameter and a slant height R or r_1 depending upon which gear is considered. To develop the surface of a cone (see Fig. 20c); the slant height r_1 is used as a radius and an arc drawn with a length at the circumference

equal to that of the circumference of the base or $S \times 3.1416$.

The developement (Fig. 20c) is of the same dimensions as the back cone of the small gear.

If then the part circumference of the development is taken as equal to the length of the pitch circle, and divided into as many spaces as the gear has teeth, and the teeth be laid off as in spur gears, then the cone formed by the development would coincide with the small gear, with the teeth of proper form and number and the base at right angles to the center line and of the right pitch diameter.

As it is unnecessary to develop more than one or two teeth on each wheel, the method shown in Fig. 20c is generally combined with the general drawing, or as shown in Fig. 20, wherein the line of centers is formed by R and r_1 and the line of contact would pass through the point of tangency of R and r_1 at an angle of $75\frac{1}{2}°$ to the line of centers formed by them.

The lines forming the tops of the teeth and usually those forming the bottom of the teeth, all converge to the apex or point of intersection of the cones.

WORM GEARING

Worm gearing is used to transmit and convert the power of a fast rotating shaft driven by a small force moving at a high velocity to a slow rotating shaft to produce a great force moving at a low velocity. It is smooth in its action and quiet. Good lubrication is essential to its life and also to reduce the loss of power due to transmission.

Worm gearing consists of a screw, the threads being on the exterior of a short cylinder (see Fig. 21) mounted

on the drive shaft. The threads are similar to the involute teeth of a rack. The screw thread engages screw threads on the edge of a disc, called the worm wheel fixed on the driven shaft in order to rotate it (see Fig. 21a).

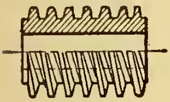


FIG. 21.

The worm wheel may be likened to the half side of a continuous nut, the threads of which are repeated aroundthe entire circumference of the worm wheel disc. By this means a continuous turning of the worm will cause also a continuous turning of the worm wheel but at a reduced ate, depending upon lead of the worm. A worm may be provided with one or more threads parallel to each other and be known as a single, double, etc., thread worm. The longitudinal distance from any point on a thread to a like point on the same thread after passing once around the worm cylinder is called the *lead*, and determines the movement of the worm wheel for one revolution of the worm. The longitudinal distance from a point on a thread to a like point on the next thread is the pitch.

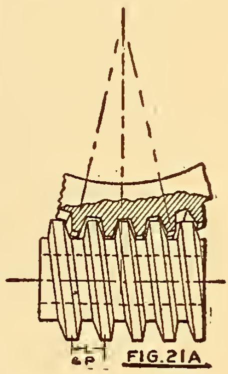


FIG. 21A.

Therefore with a single thread worm (Fig. 21c) the pitch equals the "lead" and one revolution of the worm advances the worm wheel one tooth. With a double thread worm (Fig. 21b) the pitch equals one-half the lead and for each revolution the worm wheel is advanced two teeth. With a treble thread worm the pitch equals one-third the lead and for each revolution the worm wheel is advanced three teeth. The pitch represents the distance from center to center of the teeth of the worm and worm wheel.

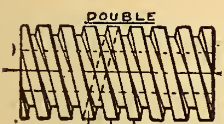


FIG. 21B

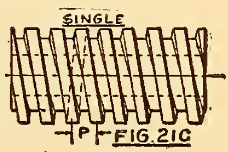


FIG. 21C

If T = number teeth in the worm wheel; n_1 = number revolutions of the worm wheel, WL = lead of worm, and WP = the pitch of the worm, both in the same unit, and n = number revolutions of the worm.

$$\text{Then } n_1 = \frac{\text{Lead } (WL) \times \text{Rev. } (n)}{\text{Teeth } (T) \times \text{Pitch } (WP)}$$

Example. A single thread worm makes 120 r.p.m. and has a pitch and lead of 1 inch. The worm wheel has 48 teeth. How many revolutions per minute will the worm wheel make?

$$n_1 = \frac{120 \times 1}{48 \times 1} = 2\tfrac{1}{2} \text{ r.p.m.}$$

Example. With the same example as above, but with a double thread worm; worm 120 r.p.m., pitch 1", lead 2", worm wheel has 48 teeth; to find r.p.m. of worm wheel?

$$n_1 = \frac{120 \times 2}{48 \times 1} = 5 \text{ r.p.m.}$$

Example—Same as above except worm has treble thread:

$$n_1 = \frac{120 \times 3}{48 \times 1} = 7\tfrac{1}{2} \text{ r.p.m.}$$

The speed ratios, in the three cases are:

120 to $2\frac{1}{2}$ or 48 r.p.m. of worm to 1 of worm wheel
120 to 5 or 24 r.p.m. of worm to 1 of worm wheel
120 to $7\frac{1}{2}$ or 16 r.p.m. of worm to 1 of worm wheel

Now, if in each case the force acting to turn the worm were the same, the force which can be exerted by the worm wheel, under the respective speeds $2\frac{1}{2}$—5—$7\frac{1}{2}$ r.p.m., would be respectively 48—24—16 times as great, neglecting friction, which plays, however, an important part in gears of this type.

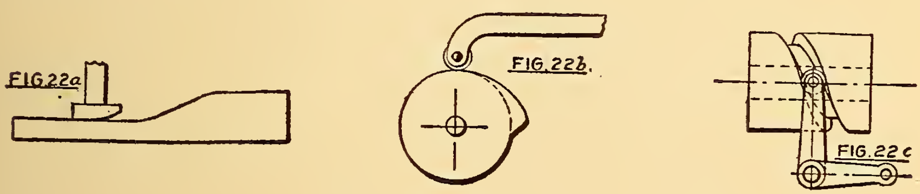
Worm gearing is extensively used in drives for elevators, automobile trucks, mechanical stokers for boiler furnaces, the guiding mechanism of traction engines. The two shafts are usually at right angles to each other and always on different planes. The worm or worm shaft must be furnished with an end thrust bearing, as the action of the worm is similar to that of a screw jack.

CAM

A cam is a device used extensively in machines, such as steam and gas engines, printing presses, automatic machinery, sewing machines, etc., for changing the direction of a motion or to produce either an intermittent or a graded motion. A cam transmits its motion, or a part of it, to a follower which is moved thereby and in the changed direction. A considerable loss from friction occurs in cam drives. The follower preferably should be provided with a roller, to engage the cam surface. The action of a cam is to engage and push the follower out of its way.

This path of movement may be straight, as in a reciprocating cam, or curved as in rotating edge cams and rotating cylinder cams. (See Fig. 22, a-b-c, respectively.)

The follower may be arranged on a slide as at *a*, or on a lever arm as in *b*, and *c*.



Types—Bar, disc and cylinder cams. Fig. 22a, represents a reciprocating bar provided with a cam projection at one side, having an inclined face to engage the shoe of the follower. At *b* (Fig. 22) an edge cam

is shown with the cam projection or incline built on the outside edge of the disc.

In Fig. 22c a cylindrical cam is shown in which, instead of the cam projection, its counterpart, a cam groove is cut into the face of the cylinder to engage the follower.

In any case the principle of the inclined plane comes into play. The force exerted being the product of weight $\times$height in feet lifted equals the foot-pounds of work—friction not considered. The approach to lifting face of the cam is usually curved to reduce the shock, incident to the meeting of the cam and follower.

EFFICIENCY

The percentage of useful work obtained from any machine or process for the known power exerted. The basis is 1 or 100 per cent—so that if 100 ft.-lbs. of power is required to do 90 ft.-lbs. of useful work, then $\frac{90}{100} = .90$ or 90 per cent, is the efficiency of the machine or process. Friction is one great obstacle to efficiency in any machine or process; lubrication therefore, in machines saves power and helps toward efficiency.

All machines and processes are more or less efficient, and the greater the useful return for power expended the higher would be the percentage of efficiency.

CHAPTER III

TRIANGLES

Their Use and Solution—Any figure bounded by three straight sides as *A-B*, *B-C*, *C-A*, joined at their ends forms a triangle. Inside the figure (see Fig. 23a) an angle is formed between each side and the side adjacent to it, or three (tri) angles *a*, *b*, *c*, as the name triangle implies. A triangle consists of six parts as follows: 3 sides *A-B*, *B-C*, *C-A* and 3 angles *a*, *b*, *c*. If three of the parts are known the other three may be found, provided one known part is a side, otherwise we could find only the proportionate relation of the sides but not their numerical value.

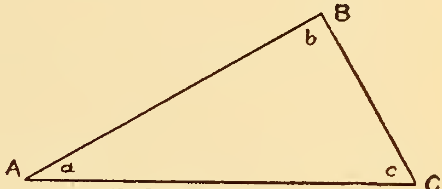


FIG. 23 A

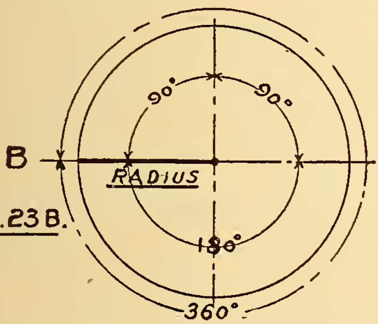


FIG. 23 B.

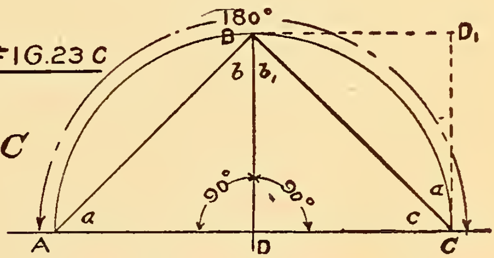


FIG. 23 C

An angle is defined by the divergence, expressed in degrees, between the two sides forming it. If any line is taken as a radius and caused to make one revolution, it would describe a circle or would have passed through 360 spaces called degrees (°). This is true whether the line be short or long(see Fig. 23B). It may also be readily observed by comparing the dial of a watch with that of a tower clock—the minute hand in each covers the 12-hour

space in one revolution. A circle, therefore, contains 360°; and for purposes of making more exact computations each degree is divided into 60 parts called minutes (') and these again subdivided into 60 parts called seconds (''), so that a circle contains 360° (degrees) or 21,600' (minutes) or 1,296,000'' (seconds).

In one-half circle (semi-circle) A to C, there are 180°, and in a quarter circle or right angle (from A to B) there are 90° (see Fig. 23C). The sum of the three angles in a triangle is equal to 180°. Proof: In the triangle A, B, C, side A-C is also the diameter of the circle with D as the center. AD, BD and DC are equal, but BD is at right angles to both, so that AB and BC must be the same length. If the left triangle ABD is folded over along the line BD, AD would cover the line DC, and AB would cover the line BC, showing that the angle: $a =$ angle c and angle $b =$ angle b'. As b and b' are equal to each other, each must equal one-half the angle formed by ABC. Again, if the triangle ABD is revolved about D, letting AD rest on BD, then BD would rest on DC and AB on BC, showing that angle a also equals b' and b equals c; but as previously found, a also equals c; therefore a also equals b. Again, the same triangle ABD, if revolved about B so that BD takes the position BD_1 parallel with DC, and AD the position D_1C parallel with BD, AB will fall on the line BC, a will be on one side and c on the other side of the line BC and the sum of the two would equal the angle DCD_1. If, however, the triangle ADB is placed over BDC with AD on DC, then BD would fall on D_1C, and as $ADB = 90°$ so will $c + a = 90°$, but a and c being equal, each will be 45°, so also $b = 45°$ and $b + b_1 = 90°$ or the total number of degrees in triangle $= 45 + 90 + 45$ or 180°.

The sum of the angles in all triangles is equal to 180°, so that if two angles of a triangle are known, their sum deducted from 180° will give the number of degrees in the third or remaining angle.

Triangles are of three kinds:

Acute angled, when the angles are less than 90°
Right angled, when one angle is 90°
Obtuse angled, when one angle is more than 90°

(See Fig. 24—a, b and c.)

In computations involving the solutions of triangles, certain relationships (called trigonometrical functions) have been established: they are based on the relationship existing between the sides and angles of right angle triangles.

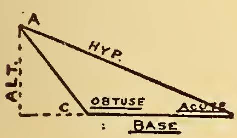


FIG. 24 A

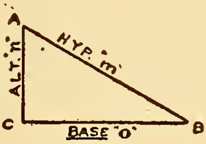


FIG. 24 B.

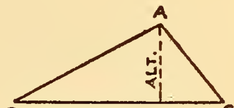


FIG. 24 C

Problems embracing right angle triangles where the lengths of two of the sides are known may be solved readily, as—

The (hypothenuse)^2 = the sum of the (base)^2 plus the (altitude)^2, or as usually expressed:

The square of the hypothenuse equals the sum of the squares of the other two sides. (See Fig. 24—b.)

The relationships of the three sides are given by the formulas:

Wherein m is the hypothenuse, o the base, and n the altitude.

and $m^2 = n^2 + o^2$

$m = \sqrt{n^2 + o^2}$, $n = \sqrt{m^2 - o^2}$ and $o = \sqrt{m^2 - n^2}$,

Example. $m = 5$, $n = 3$, angle $c = 90°$. To find o.

Solution. $o^2 = m^2 - n^2$ or $o^2 = (5 \times 5) - (3 \times 3) = 25 - 9 = 16$.

$o = \sqrt{16} = 4$.

FUNCTIONS OF RIGHT ANGLE TRIANGLES

(See Fig. 25.) They are known as follows:

Sine (abbreviation *sin*)—The side opposite the angle when the hypothenuse is considered as the radius of the circle.

Cosine (abbreviation *cos*)—The side adjacent to the angle when the hypothenuse is considered as the radius of the circle. It is the *sine* of the complementary angle, i. e., the *sine* of the angle remaining, after deducting the angle, first mentioned from 90°.

Tangent (abbreviation *tan*)—The side opposite the angle when the base is considered as the radius of the circle.

Cotangent (abbreviation *cot*)—The side opposite to the complementary angle with the base as the radius of the circle.

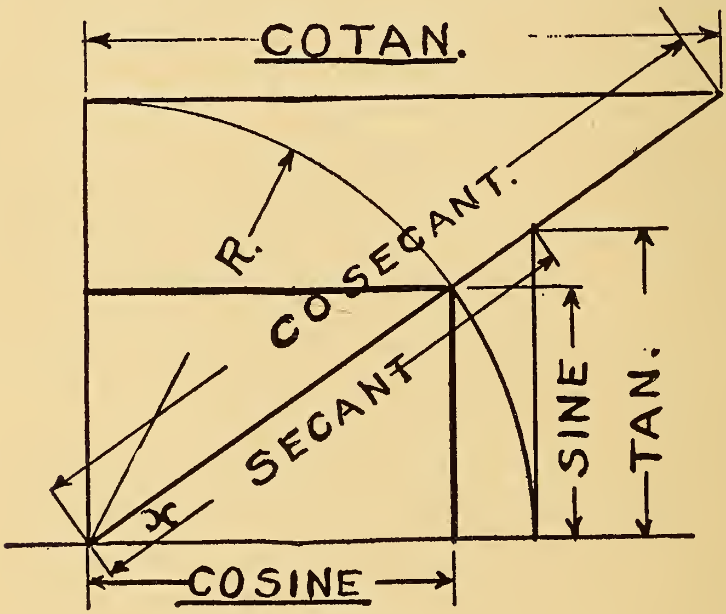


FIG. 25.

Secant (abbreviation *sec*)—The hypothenuse of the triangle when the base is considered as the radius of the circle.

Cosecant (abbreviation *Cosec*)—The hypothenuse of the complementary angle when the base is considered as the radius of the circle.

The sines and cosines of angles lie wholly within the arc of the circle.

The tangents and cotangents of angles lie wholly without the circle and are tangent thereto as the names imply.

The hypothenuse of the triangle, when dealing with sines and cosines, acts as the radius of the circle, but when dealing with tangents and cotangents it is the line which connects either the end of the tangent or the end of the cotangent with the center of the circle, with the base of the triangle considered as the radius. In either case part of the secant or cosecant lies within and the balance without the circle.

The functions of angles indicate the relationship or ratio between the sides of right angle triangles and by their use the actual lengths of the sides may be found, provided three parts of the triangle are known, one of which is a side.

Tables of the functions of right angle triangles, called "Tables of Trigonometric Function," may be found in the various handbooks. In order that the student may understand how to apply such tables and how the approximate values may be found, attention is called to Fig. 25A, etc, in which let, *R* indicate a radius of any length, and its value assumed as the whole number one (1) or 100%. Now by drawing an arc of 90° and forming a right angle between the two extreme positions the percentage value of sines and cosines may be readily obtained, if the arc be divided into 90 equal parts each representing a degree (°) and the two sides, base and

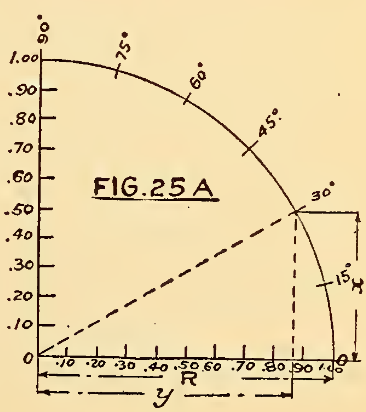


FIG. 25 A

altitude, each be divided into 100 equal parts, each part being 1–100 or 1% of the whole or radius (1). In the diagram for the sake of clearness only the 15° intervals and the 10% distances are given. For accurate computations, the trigonometric tables only should be used.

Assuming as an example (see Fig. 25B) a right angle triangle with the hypothenuse equal to 15'' and the angle confined between it and the base, *a*, be 30° and the angle between the base, *a*, and the altitude, *b*, of course 90°. To find the respective lengths of the side opposite, or *b*, and the side adjacent, or *a*, to the 30° angle.

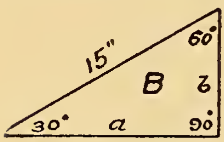


FIG. 25B.

Now, by referring to the diagram, *R* would represent the hypothenuse, or 15'', moved to the position indicated by the broken line and meeting the arc at the 30° division, the side opposite and the side adjacent to the 30° angle would bear the same percentage relationship to the hypothenuse and to each other as the base and altitude of the given triangle bear to the 15'' hypothenuse and to each other. If now the altitude *x*, be measured or read, its length will be found to be equal to 50 parts, or 50%, of the hypothenuse, and in the case of the 15'' hypothenuse with the 30° angle the altitude would be 50%, or one-half of 15'', or $7\frac{1}{2}$''. If, in like manner, the distance *y*, side adjacent be measured, it would be found to be nearly 87% (actually .86603) of the radius or hypothenuse, and if the 15'' be multiplied by .86603 the product or 12.9905'' would be the length of the base or side adjacent to the angle of 30°. The same method would have to be used for any angle when considering sines and cosines; the sine being the side opposite or altitude, and the cosine the side adjacent or base of the triangle. In the above

example two of the angles were known, i. e., 30° and 90°, total 120°, have been accounted for; by subtracting this sum from 180° the value of the third angle or 60° also is found. The value of all three sides and all three angles now are known. The 60° angle in this case is the complementary angle, as it represents the difference between the 90° and 30°. If the triangle were turned so that the side, y, represented the altitude, it would still be of the same length, but would be opposite the angle of 60° and therefore be the sine of 60°; in like manner, the side, x, would be the cosine of 60°. It will now be understood that a side of a triangle may be considered either as a sine or as a cosine, depending which of the two sides of the right angle is taken as the base. It will also be seen that the sine of an angle is equal in value to the cosine of the complementary angle, and *vice versa*; therefore the affixing the co to sine to indicate the sine of the complementary angle.

By examining a table of natural sines and cosines, it will be observed that the sines increase from 0 to 100% or 1, while the cosines decrease from 1 or 100% to 0, in passing around the arc from 0° to 90°.

The tables usually are so arranged that when finding a sine you follow the degree readings in the column (1st col.) and the sine column (2nd col.) from the top downward to 45°; then jump to the 3rd column for sines, and the last column for the degrees, reading upward until 90° is reached. Tables are more complete and more accurate than the diagram, and therefore are used in preference to the diagram.

In cases where the sides of the right angle triangle are given and it is desired to find the angles confined between the sides, it is necessary only to divide the side opposite

the angle to be obtained by the hypothenuse and find the angle in the table under sines corresponding to the decimal obtained from such division. If, on the other hand, the side adjacent had been taken as the dividend, then the decimal equivalent would have to be sought for under the head of cosines in the table and the angle in degrees, etc., taken found opposite thereto.

TANGENTS AND COTANGENTS

These are used relatively in the same manner as the sines and cosines, but differ in magnitude due to being wholly without and tangent to the arc.

SECANTS AND COSECANTS

When solving right angle triangle on the basis of tangents and cotangents the secant and cosecant represent the hypothenuse, as already stated. The prefix co is used for the same purpose as with sines and tangents.

CHAPTER IV

FLUIDS

Those substances such as gases and liquids whose particles have little or no attraction for each other and may be easily separated. All fluids have weight; they tend to seek their level and are affected by heat and cold. The gases, for example air and steam, may be compressed by pressure, and likewise will expand with a reduction in pressure. The liquids, for example oil and water, are relatively incompressible. Air and water are the most widely known fluids.

Air, subject to excessive pressure and cooled, may be changed from a gas to a liquid (liquid air).

Water readily assumes three states, due to changes in temperatures:

Gaseous (steam) above 212° F.
Liquid (water) between 212° and 32° F.
Solid (ice) below 32° F.

It may be possible that air, too, would assume a solid state if the pressure were sufficient and the temperature low enough.

The whole earth's surface is covered by a blanket of air, which, due to its weight, exerts a pressure of 14.7 pounds on each square inch of surface at sea level. On the tops of high mountains the pressure is correspondingly less, due to a less thickness of the blanket.

Water (fresh) weighs approximately 62.5 lbs. per cu. ft.

Water (salt) weighs approximately 64 lbs. per cu. ft.

As a cubic foot contains 1728 cu. in., then $62.5 \div 1728 =$ weight of 1 cu. in. or a 1'' cube, and the pressure (weight) exerted by a vertical column 1'' sq. in. $\times$ 12 inches high would be $\frac{62.5 \times 12}{1728} = 62.5 \div 144 = .434$ lbs., nearly, pressure per sq. in. for every ft. in height.

As air exerts a pressure of 14.7 lbs. per sq. in. at sea level, then 14.7 ÷ .434 = 34 ft., the height of a column of water which balances approximately the atmospheric pressure.

Fluids have different weights and deusities, which explains why oil floats on water, also why a balloon filled with a gas lighter than air rises—also why heated air rises and the cause of a draft in a chimney.

Fluids when confined under pressure, whether due to weight pressing down or otherwise, press against all sides uniformly at right angles to such surface and for each square inch thereof. Solids press only in the direction of the applied force producing the pressure. A block of ice below freezing temperature would, like wood or stone, transmit an applied force in a direct line to the surface upon which it rests without outside aid, due to the strong attraction of its particles (molecules) for each other.

If the temperature be raised above freezing, the ice would melt and run off in the form of water and allow the weight to settle, because in melting the strong molecular attraction is destroyed and the molecules try to get from under the load and seek their level, whether the load be an extraneous weight or superimposed ice or water. If, however, the block of ice had been fitted neatly into a vessel or cylinder and then melted, the walls of the vessel would prevent the water from running off and hold the molecules to their work against their normal desire to get away in any direction, if free to move upon each other. If the cylinder, *A*, as in Fig. 26, had been provided with a neatly fitted sliding piston resting on the water or melted ice and supported a weight, *w*, the force due to such weight would

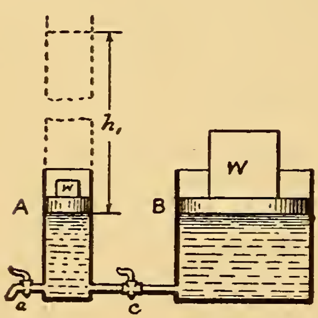


FIG. 26

be distributed evenly over the entire water surface and the downward pressure on each square inch would be resisted by an equal upward pressure per square inch under the law of balanced forces. The molecules forming the water, being restrained by the cylinder against movement, transmit the force from one to another in all directions alike and finally to the cylinder walls and at right angles thereto, which react with the same intensity per unit of surface as the force due to the weight applied.

Had the piston been made very loose fitting, so that the water could readily pass between it and the cylinder, a condition similar to that of the non-restrained melting ice would occur, because the molecules could get from under the load and be pushed up alongside the piston and allow it to settle or sink. The opening of a valve or stop-cock, as at *a*, would produce a like result.

If, in Fig. 26, a second cylinder, *B*, with a neat fitting piston be connected with the first by a pipe and valve, *c*, and said valve be opened, the pressure per unit area in the two cylinders would be equal, but the total pressure or weight supported would be to each other as their respective areas, or as to the squares of their diameters.

Example. A weight of 50 lbs. is on the small piston of 2'' diameter; what weight will a large piston 6'' diameter sustain if the cylinders are connected?

Let. d —diameter small piston
and d_1 —diameter large piston
W —weight on small piston
W_1 —weight which can be supported by large piston

$$\text{Then } W_1 = \frac{W \times d_1^2}{d^2} = \frac{50 \times 36}{4} = \frac{1800}{4} = 450 \text{ lbs.}$$

If weight, W_1, is to be lifted 1 inch in the above example, the weight, W, on small piston would have to descend

9 inches (i. e., ratio of pistons)—as the inch-pounds of work are the same in either case.

$$W \times 9'' = W_1 \times 1''$$

$$50 \text{ lbs.} \times 9'' = 450 \text{ lbs.} \times 1''$$

The above is the principle of the hydrostatic press for exerting very heavy pressures over limited distances. In the place of one long stroke of the small piston, the small cylinder is arranged as a force pump.

If both pistons were removed from the cylinders, the water columns would balance each other and their surfaces be level. Hence the expression "water seeks its level." The vertical distance from the center of the connecting pipe to the surface of the water or the head, h, would be the same, and if said pipe were 1 square inch in area (approximately $1\frac{1}{8}''$dia.) and the head, $h=5$ feet, then the opposing pressures, P, in said pipe in lbs. per square inch from each cylinder would be,

$P = h$ in feet$\times$.434 (weight of column of water 1 foot high and 1 square inch in cross section)

or $P = 5 \times .434 = 2.17$ lbs. per sq. inch

In Fig. 26, if the small cylinder A were extended, as indicated by the broken lines, and the weight and piston removed and replaced by 50 lbs. of water the large weight (450 lbs.) would be balanced, as before; the water, surface in the small cylinder would, however, be proportionately higher than in large cylinder. This height may be found as follows:

If pipe$=2''$; its area equals 3.1416 square inches and W = weight of water equals 50 lbs.

Then $50 \div 3.1416$ = lbs. water for each sq. in. of surface but .434 lbs. equals weight of 1 sq. in. column of water 1 ft. high, so that the height (h_1) is

$$W \div \textit{diam.}^2 \times .7854 \div .434 = h_1$$

$$\text{or} \quad 50 \div 3.1416 \div .434 = 36.7 \text{ ft.}$$

In tanks the shape determines only the volume and the consequent total weight to be supported. The height of water or head from the bottom of the tank to the water surface determines the pressure on each unit of surface of the bottom, and is independent of the volume. The unit pressure on any intermediate plane will be proportional to its depth below the water surface.

Figs. 27-A-B-C-D-E show tanks of various shapes, in which d is the large diameter and d_1 the small diameter; h is the total height in feet from the bottom to the water surface, and h_1 the height in feet from the water surface to b, b_1.

The volume of A is the largest, the vol. $B =$ vol. C, and vol. $D =$ vol. E, but each is less than A, and likewise the weight of the water contained is less.

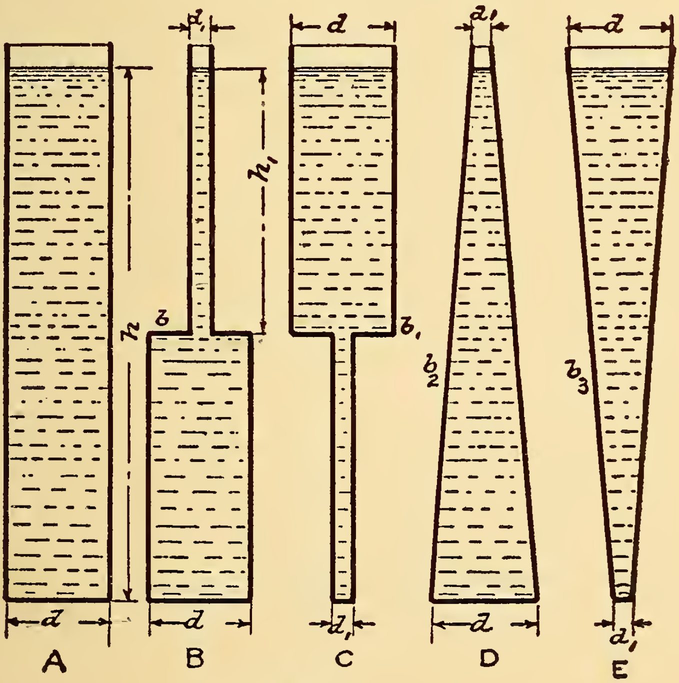


FIG. 27.

The diameters, d, are equal in each case; so, too, the diameters, d_1, are equal to each other.

The head, h, in feet is the same in each case
Also the head, h_1, in feet is the same in each case.

If P represents the unit pressure in lbs. per square inch at the bottom and P_1 the unit pressure in lbs. per square inch at the point b, b_1,

Then $P=h\times.434$ lbs.
and $P_1=h_1\times.434$ lbs.

Example. If $h=5$ ft. and $h_1=2.5$ ft.
$d=12''$ diameter (113.1 sq. in. area)
$d_1=1\frac{1}{8}''=$dia. (1 sq. in. area).

Find the weight of water contained; the unit pressure per square inch and total pressure on the extreme bottom in A, B, C, D and E, respectively.

Also to find unit pressure and direction on b, b and b_1b_1 of B and C, respectively.

A contains 6786 cu. in. $\div$ 1728 cu. in. per cu. ft. $\times$ 62.5 lbs. per cu. ft. $=$ 245.4 lbs.

B and C each contain for

$$\left.\begin{array}{l}\text{large part } \tfrac{1}{2} \text{ of 245.4 lbs. or 122.7 lbs.}\\ \text{small part 54 cu. in.} \div 1728 \times 62.5 \text{ or 1.95 lbs.}\end{array}\right\} \text{124.7 lbs.}$$

D and E each contain 2466.2 cu. in $\div$ 1728 $\times$ 62.5 $=$ 89.2 lbs.

The pressure per square inch on the bottom of A, B, C, D, E, respectively, is 5 ft. $\times$ .434 lbs. per sq. in. for each ft. $=$ 2.17 lbs.

and the total pressure on the bottom of

A	$=113.1\square''$	$\times 2.17$	$=245.4$	lbs.	pressure
B	$=113.1$	$\times 2.17$	$=245.4$	"	"
C	$=1.$	$\times 2.17$	$=2.17$	"	"
D	$=113.1$	$\times 2.17$	$=245.4$	"	"
E	$=1.$	$\times 2.17$	$=2.17$	"	"

The unit pressure per square inch on b, b, and b_1, b_1 is $2.5' \times .434$ or 1.085 lbs., and the total pressure outside the small column

$$113.1\square'' - 1\square'' = 112.1\square'' \times 1.085 = 121.53 \text{ lbs.}$$

In the case of B the pressure is upward, and in C the pressure or weight is downward.

In B and D the total pressure on the bottom is the same as in A, due to the reactions of b and b_2, and full head, h, as in A. The walls hold the top and bottom plates from separating.

In C and E the total pressure at extreme bottom is the same, due to the upward reactions of b_1 and b_3, respectively.

BUOYANCY

The relative lightness of two substances, one or both of which are fluids. The lighter substance is buoyed up and floats, and any substance tends to float in the fluid.

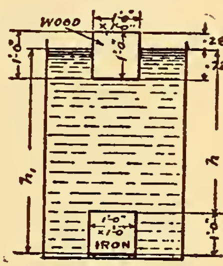


FIG. 28.

A block of wood 1 cu. ft. or 45 lbs. will float in water, a part being above and the balance below the water surface. A block of iron 1 cu. ft. or 450 lbs., however, will sink to the bottom. (See Fig. 28.)

The weight of water displaced by that portion of the wood block below the surface will equal the total weight of the block of 45 lbs., which is only 72% ($45 \div 62.5$) of the weight of an equal volume of water. Each, however, has the same area and height, so that for the same weight the wood block would be immersed only 72% of its height or .72 ft. (8.64 inches).

The block pushes down with a force of 45 lbs. on 144 sq. in. of surface, and must be pushed up with the same force, or $45 \div 144 = .3125$ lbs. per sq. in. Water *per ft.*

height exerts .434 lbs. pressure per sq. in. (see water, under fluids). Then $.3125 \div .434 = .72$ ft. (8.64 inches), the depth the block is immersed to provide the requisite pressure of .3125 per sq. in. to support it.

Now, the iron block being totally immersed with water above, it is pressed down by a force equal to h (head in ft.)$\times .434$ lbs. per sq. in.$\times$area (144 sq. in.), but is also pushed up from underneath by a force $h_1 \times .434 \times 144$ sq. in.

It will be seen that, due to the greater head, the force from underneath will be the larger and tend to lift the block—the difference in their unit weights (62.5 water and 450 lbs. iron), however, is too great.

Example. A block of cast iron $12'' \times 12'' \times 12''$ weighing 450 lbs. is placed in a vessel of water. The depth from water surface to bottom is 4'–0''. How much greater is the force pushing up than that pushing down?

Solution. $4'\text{–}0'' \times .434$ lbs. per sq. in $= 1.736$ lbs. pressure per sq. in upward; $3'\text{–}0'' \times .434$ lbs. per sq. in $=$ 1.302 lbs. pressure per sq. in. downward, or .434 lbs. less; then 144 sq. in.$\times .434 = 62.496$ lbs. excess of pressure upward—but this equals the weight of the 1 cu. ft. of water displaced by the 1 cu. ft. of iron.

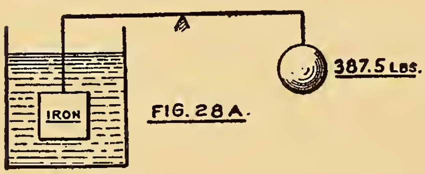


FIG. 28A.

Now, if the block of iron were weighed in the water, as shown in Fig. 28A, it would weight $450 - 62.5$ lbs., or 387.5 lbs.

It may be shown that the shape of the block or the depth to which it is *sunk* does not alter the above if the volume remain the same, because for each cubic foot

of iron or part thereof a like volume of water would be displaced and the buoyancy per unit of weight remain the same.

If the volume of the immersed substance be increased, the weight remaining the same, the upward tendency would increase in proportion to the increased displacement. In like manner, if the volume remain the same but the weight be decreased, the upward tendency would increase.

Example. Will a hollow cast-iron cube $12'' \times 12'' \times 12''$ with $\frac{1}{4}$ inch thickness of wall float in water if cast iron weighs .2607 lbs. per cu. in.?

Solution. 1 cu. ft. of water weighs 62.5 lbs.

Cube—4 sides $\times 11\frac{3}{4}''$ wide $=$

$47 \times 11\frac{1}{2}''$ high $= 540.5$ sq. in.

Top and bottom $12'' \times 12'' =$

$144 \times 2 = \underline{288.\ \text{ sq. in.}}$

$825.5 \times \frac{1}{4}'' = 207.1$ cu. in.

$207.1 \times .2607$ lbs. per cu. in. $= 53.991$ lbs. weight of cast-iron hollow cube.

62.5 lbs. weight of cube of water $-$ 54 lbs. weight of cube of iron $= 8.5$ lbs. iron cube is lighter than displaced water.

The cast-iron cube will float, therefore, and would sustain approximately $8\frac{1}{2}$ lbs. more than its own weight.

It will now be seen that steel vessels—even though steel is heavier than water—can be constructed to be capable of displacing an amount of water exceeding their own weight, and thus be suitable for cargo purposes.

PUMPS

Machines for raising water or other fluids from either a low elevation or a low pressure to a higher elevation or pressure, and usually understood to refer to those machines wherein one or more movable members (pistons, plungers or impellers) act directly on the fluid to be moved.

Fluids (gases and liquids) always flow from a state of high pressure to a lower one.

Pumps are known as of two general types:

Lift Pumps—Those for lifting water usually under atmospheric conditions from wells or cisterns to the surface of the ground or operating platform. (See Fig. 29A.)

Force Pumps—Those whereby the water is discharged against considerable pressure, whether due to differences in elevation or differences in pressure, as in feeding steam boilers. (See Fig. 29B.)

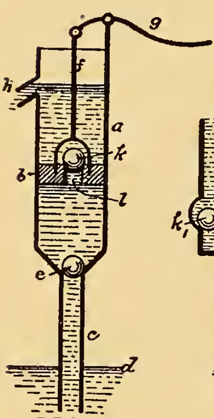


FIG. 29A

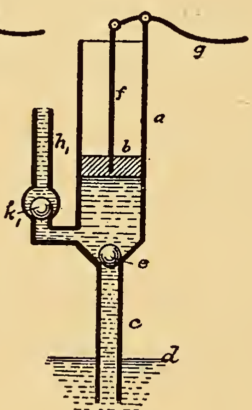


FIG. 29B.

In Fig. 29A a simple lift pump is shown, and in Fig. 29B a simple force pump is shown. Each consists of a pump cylinder, *a*; a reciprocating piston or plunger, *b*; a suction pipe, *c*, leading to a point below the water surface, *d*; a suction valve, *e*, to permit water to flow from the suction pipe to the cylinder, but not to return; a plunger rod, *f*, fixed to the plunger at one end and pivoted to an operating lever, *g*, at the other end; the discharge outlets, *h*, h_1, respectively—the one *h*, at the top of the lift pump cylinder and above the

plunger; the other, h_1, at the bottom of the force pump cylinder and below the plunger; discharge valves, k, k_1, respectively, are provided—the one, k, for the lift pump, and the other, k_1, for the force pump; a passage, l, through the lift pump plunger is controlled by the valve, k, which permits the water to flow upward through the plunger but not return.

It will be noticed that each type has the same number of parts, the differences being only in the discharge outlets and valves, and that one plunger has a passage through it while the other is solid.

The plungers of pumps are arranged to slide freely in the cylinders, but sufficiently tight to prevent water from passing between them and the cylinder walls.

Pumps are provided for raising water or fluids and the work they do in foot-pounds is equal to the weight raised by the height in feet through which it is raised; they form only a more convenient method for doing the work.

If, for example, it be desired to lift 500 gallons of water at $8\frac{1}{3}$ lbs. per gallon from a cistern or tank to an elevated tank 27 feet above the cistern, the work to be done would be $500 \times 8\frac{1}{3} \times 27$ ft. $= 112{,}500$ ft.-lbs. of work, and could be performed with a bucket alternately filled at the cistern and emptied into the elevated tank; the number of trips from the one to the other would be determined by the capacity of the bucket.

The plunger of the lift pump is no other than the bucket fitted to a tube and having a bottom hole provided with a check valve to facilitate filling. The bucket still must travel the full height for each delivery, and the tube, of course, extend the full height, also from below the water surface of the cistern to a point above the tank.

It is apparent that before the bucket starts upward on its first trip the water level will be the same inside and

outside the tube, likewise the pressure (atmospheric air 14.7 lbs.) inside and outside will be alike and the bucket be flooded. Now, with any upward movement of the bucket it tends to create a vacuum (space devoid of air) and to destroy the atmospheric pressure (14.7 lbs.) above the water surface inside the tube, while the pressure on the water surface outside the tube has not been affected. Due to the unbalanced air pressure, the water will continue to follow the bucket upward until the pressure of the water column balances the difference in pressure inside and outside the tube.

As shown, however, in a previous chapter the atmospheric pressure (14.7 lbs.) should support a column of water 34 feet—in practice, no more than 25 feet to 28 feet is attainable due to leaks and other causes.

If, now, the lower end of the tube be arranged with a suction valve, so the water could continue to follow the bucket or piston upward but not return, the simple lift pump would be complete, only the stroke would be excessive and impractical; by cutting off 20' to 25' of the lower end of tube (pump cylinder) and substituting a suction pipe, a short stroke only of the piston or plunger would be necessary.

In the force pump with the first downward stroke of the solid piston the air is forced out through the discharge valve and passage, and the following upstroke tends to create a vacuum which permits the pump cylinder to fill with water, which is forced out through the discharge on next down stroke. The force pump and its arrangement of discharge valve, k_1, permits of piping being connected to the discharge outlet and the water to be discharged under a considerably greater pressure than that due to the elevation of the pump; for, unlike the lift pump, which is

limited to approximately 25' to 28' lift, its limit of pressure on the discharge is governed by the physical strength of the pump and the available power behind it.

The lift pump is used principally for pumping out cisterns, tanks and ditches.

The force pump is used for any purpose where the water or fluid is to be distributed through pipes under pressure, such as boiler feeding, water supply and the like.

The **Centrifugal** pump, unlike the (reciprocating) lift and force pumps considered above, has no valves and no plungers, but has in place of the plunger an impeller wheel, driven at high speed and having vanes or spokes which impart centrifugal force to the water passing through the impeller. The water enters the pump casing at the center of the shaft and leaves it at the circumference.

Centrifugal pumps must either be below the water surface or be primed (filled) before starting, as they would only churn the air and not establish the difference in pressure between the inside and outside of the suction pipe.

They are in many cases supplanting reciprocating pumps due to their simplicity of construction, their simplicity of operation and the small amount of attention required.

CHAPTER V

STRENGTH OF MATERIALS

In the design and construction of machines or the several parts or elements of which they are composed it is necessary not only to apply the principles of mechanics as to the forces involved and their action but to also select the proper material and its size and shape to safely transmit such forces. It is evident, therefore, that the strength and nature of the materials to be used and their adaptability should be known.

A force, depending upon its nature, acting on any one element of a machine may tend: (See Fig. 1.)

To pull such member apart by means of a tensile strain,

To crush it under a compressive strain, or

To cut it in two under a shearing strain.

The external or applied force tends to change the shape or to disrupt the part and subjects it to a "strain," as stated above; the force acting within the body to resist disruption is called "stress."

There are, therefore, three kinds of simple or direct stress:

Tensile—to resist pulling apart, as in ropes and tie-rods.

Compressive—to resist crushing, as in columns and supports.

Shearing—to resist being cut in two, as in cutting or punching materials or plates.

A tensile strain causes elongation.

A compressive strain, pushing together or compression.

A shearing strain, cutting across to separate the particles of the material.

Any applied force tends to deform the body or member and any one or all three kinds of stress may act to resist such deformation.

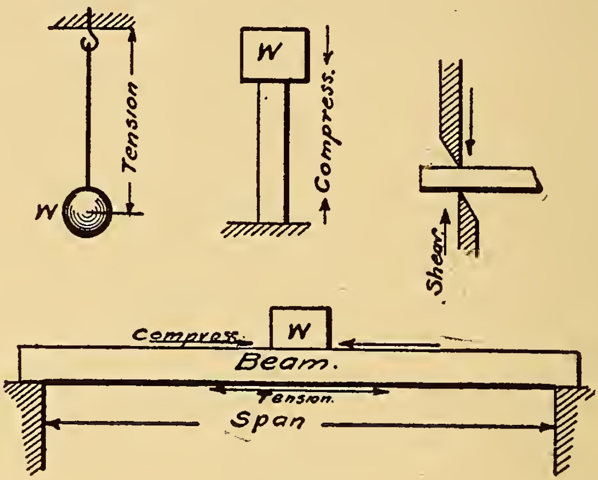


FIG. 1.

UNIT STRESS

When a body is subjected to a strain, the total stress inside the body is considered as distributed evenly over the entire cross-sectional area. The total stress divided by the said area in square inches or square feet is called the unit stress per square inch or per square foot, according to the terms taken for the cross-sectional area.

If A equals the total area, P—the total stress in pounds and S—the unit stress; then

$$A=\frac{P}{S}, \qquad P=AS, \qquad S=\frac{P}{A}$$

The unit stress for metals and timber is considered usually in pounds per quare inch, while for brick and stone and concrete it is taken in pounds per square foot.

Any material used in construction has a certain rigidity due to the attraction of its particles for each

other; this mutual attraction holds the material to its shape and determines its strength against deformation when strained. The various materials used in construction vary in strength and suitability due to differences in their nature—some being suitable for use under compression but unsuitable under tension.

HEAT

Heat affects the strength of metals, as it tends to soften them and to destroy the strong attraction between the particles. This may be observed by taking an iron bar which, when cold, offers considerable resistance to bending but upon being heated bends under its own weight. Metals of the same kind vary in strength, due to differences and imperfections in manufacture; timbers, etc., to their growth and treatment. Therefore, average values for strength of a material are usually given in the tables of hand books, etc.

ULTIMATE STRENGTH

When a material is subjected to a gradually increasing strain or load and consequent stress until it is ruptured, the intensity of such strain in pounds divided by the area of the cross-section in square inches is called the ultimate strength per square inch, and may be tensile, compressive or shearing, depending how the load was applied. The ultimate strength may be, and probably is, reached just before rupture takes place.

In the design of structures the ultimate strength or resistance against rupture should not be taken as the stress to which the material may be subjected. In practice the allowable stress is taken as about 30 to 50 per cent. of the elastic limit of the material or to a fractional

part of the ultimate strength, such fractional part being called factor of safety.

ELASTIC LIMIT

All materials are more or less elastic and will return to their original shape after being deformed by the action of a load, provided such load has not been great enough to produce a permanent set or deformation. The maximum load per unit of area to which a material may be subjected and still return to its original shape—i. e., no permanent set has taken place—is known as the elastic limit of the material.

FACTOR OF SAFETY

Often in place of using the elastic limit for purposes of computation a fraction of the ultimate strength of the material varying from one-third to one-twentieth of such ultimate strength is taken; depending on the condition of applying the load and the nature of the material.

MACHINES AND STRUCTURES

Are subjected to various conditions of loading: Steady, intermittent, shocks, etc.; judgment, therefore, must be exercised in assuming the working stress or factor of safety to be used in any special case.

DEFORMATION

All materials are more or less deformed when subjected to a load, but immediately assume their original form, provided the stress produced was well within the elastic limit of the material. Under this condition the deformation is practically proportional to the stress and also proportional to the length of the bar.

COEFFICIENT OR MODULUS OF ELASTICITY

The ratio between the stress per unit of area to the unit deformation when such stress is within the elastic limit.

If E be the coefficient of elasticity, S the unit stress and d the unit deformation, then within the elastic limit,

$$E=\frac{S}{d}, \quad \text{and} \quad S=E\times d$$

If the elastic limit is exceeded, a permanent set or deformation results and the material does not return to its original shape or form.

WORKING STRESS

The allowable stress to which a material may be subjected without injury. It should be more than the weight of the member and the extraneous load applied thereto. A bar if sufficiently long would bend or break, due to its own weight, without any extraneous load.

TENSILE STRENGTH

The ability of a material to resist a tensile strain, or to being pulled apart, by an equal tensile stress. If such stress is well within the elastic limit, no injury to the material ensues and no permanent set takes place.

Any one of a number of materials might be used to support a weight suspended thereby, but each would have to be of a cross-sectional area commensurate with its strength per square inch (unit area).

STRENGTH OF MATERIALS—TENSILE

	Average in lbs. per Sq. In.		Coefficient of Elasticity
	Ultimate	Elastic Limit	
Timber.....	10,000	3,000	1,500,000
Cast Iron...	20,000	6,000	15,000,000
Wrought Iron	48,000	24,000	25,000,000
Steel.......	60,000	30,000	29,000,000

A working stress about .5 (elas. limit) is used, if the load is steady and slowly applied. If the material is subject to sudden loading or shocks, one-half or one-third of the above should not be exceeded.

It will be seen with the above working stresses, a bar of cast iron of one square inch area is only one-fourth as strong as a similar bar of wrought iron, but approximately twice as strong as a similar bar or rod of wood.

If L = the weight in pounds to be supported,
a = the cross-sectional area of the bar in sq. in., and
S = allowable working stress in pounds per sq. in.,

Then $a=\frac{L}{S}$, $\qquad S=\frac{L}{a}$, $\qquad L=a\times S$

Example. What weight (steady) will a round steel bar 2" in diameter support safely?

Solution. $L=a\times S$. The area equals $2''\times 2''\times .7854=$ 3.1416 sq. in. $S=16{,}000$ lbs. $L=3.1416\times 16{,}000$ or 50,266 lbs.

Example. A weight of 48,000 lbs. is to be suspended by a square wrought iron bar. What size of bar is necessary if the load is steady?

Solution. $a = \frac{L}{S}$, L = 48,000 lbs., $S = 12,000$ lbs.

Then $a = 48{,}000 \div 12{,}000 = 4$ sq. in. or $2'' \times 2''$ in size. If a round bar had been called for, the diameter of a circle whose area is 4 sq. in. would have to be found.

Example. To a suspended flat steel bar $2'' \times 1.5''$ a weight of 24,000 lbs. is suddenly applied. What is the unit stress to which the material is subjected and is the bar safe?

Solution. $S = \frac{L}{a}$, $L = 24{,}000$ lbs. $a = 2'' \times 1.5'' = 3$ sq. in.

Then $S = 24{,}000 \div 3 = 8{,}000$ lbs. unit stress to which the bar is subjected. Under this loading one-half the working stress of 16,000 lbs. is taken, and as this allows 8,000 lbs. per sq. in. the bar is safe.

In round tension rods and bolts where the load is applied through the medium of screw threads the cross-sectional area of such bars is taken at the root of the thread, as this area determines the strength of the bar or bolt. In long tie-rods the ends are usually upset to provide a sufficiently larger outside diameter, so that the diameter at the root of the thread and the diameter of the bar between the upset ends are about equal.

In bolts the height of the nut is usually equal to the outside diameter of the threaded portion and the height of the head somewhat more than two-thirds the diameter. These proportions are used in order to have all parts equally strong approximately.

Under the heading "Factor of Safety" the proper value of such factor for any special case was not considered; such, however, may be found in the accompanying table taken from "Machinery's" hand book.

FACTORS OF SAFETY

	Steady Load	Load Varying from 0 to Max.		Sudden Varying Loads and Shocks
		One Direction	Both Directions	
Wood..........	8	10	15	20
Cast Iron.......	6	10	15	20
Wrought Iron...	4	6	8	12
Steel...........	5	6	8	12

In the table the factors are given as whole numbers. It is, however, to be understood that the ultimate strength of the material is to be divided by the proper factor to find the allowable working stress; after which the method of solution is the same as if the allowable working stress based on the elastic limit had been taken directly from a table of strengths of materials.

ROPES AND CHAINS

These are used in tension; the student is referred to any of the various engineers' pocketbooks, which have tables giving the strength of different kinds of ropes, chains, cables, etc.

COMPRESSIVE STRENGTH

As compression is the opposite of tension, the compressive strength represents the ability of a material to resist the crushing action of a load acting thereon. When the length of a piece is short compared with its cross-sectional area, the material is considered as in direct compression. When the length is ten or more times the smallest cross

dimension, it is liable to fail, due to bending. Materials when in direct compression have approximately the same characteristics of the elastic limit and coefficient of elasticity as in tension.

The average ultimate compression strengths, however, are taken as follows:

Timber (lengthwise)	8,000 lbs. per sq. inch
Cast Iron	80,000 " " " "
Wrought Iron	55,000 " " " "
Steel (castings)	100,000 " " " "
Brick	6,000 to 10,000 " " " "
Stone	10,000 to 12,000 " " " "

The formulas for direct compression are the same as those for tension.

SHEARING STRENGTH

The ability of a material to resist being cut in two when acted upon from opposite sides by two members of a machine or structure tending to slide upon each other, as in the blades of scissors, shears, punches and dies, plates in tension, as in riveted joints, the stripping of the threads between a nut and its bolt, etc.

The formula for direct shear are the same as for tension and compression. L, representing the weight; a, the cross-sectional area in shear; and S, the allowable shearing stress of the material.

In riveted joints, where rivets prevent the sliding of the one plate upon the other, the area of cross-section of the rivet times the allowable shearing stress gives the shearing value of the rivet. In punching or in the shearing of plates the area to be sheared is equal to the length of the sheared portion multiplied by the thickness of the plate, both being in inches.

Some materials, such as wood and similar fibrous materials, offer a greater resistance to shear when cutting across the grain than when cutting parallel with it.

The average ultimate shearing strengths of various materials are taken as follows:

Timber, parallel with the grain	600	lbs. per sq. in.
Timber, across the grain.....	2,500	" " " "
Cast Iron.........	18,000 to 20,000	" " " "
Wrought Iron.............	40,000	" " " "
Steel.............	70,000 to 75,000	" " " "

TORSION

Any action which tends to twist a member or a part of a machine puts such part under a torsional strain; this must be counteracted by an equal torsional stress set up in such part. If the unit stress is within the elastic limit of the material no permanent deformation results, but if in excess of such limit a permanent deformation and possible disruption of the part results. Many of the common tools, such as screw-drivers, drills, socket wrenches, keys for locks, shafts for transmitting motion and power, etc., in their use are subjected to torsional strain.

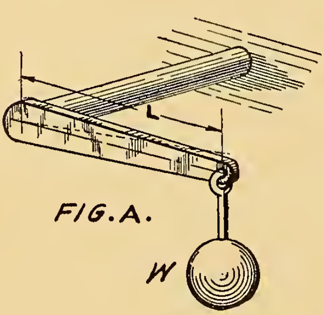


FIG. A.

A shaft or tool assumed as fixed at one end and provided with a lever or crank arm to be acted upon by a weight or a pull (see Fig. A) would be subjected to a torsional strain or twisting moment.

If L represents the lever arm of the shaft in inches, W the weight or pull in pounds, and $T.M.$ the twisting moment in inch-pounds, then $T.M. = W \times L$.

Any weight on the lever arm will twist the shaft more or less, starting with zero at the fixed end and reaching a maximum where the lever is attached. The amount of the twist increases in proportion to the length of the shaft, provided the elastic limit is not exceeded. The lever will return to its original position also after the load is removed if the elastic limit was not exceeded.

The resultant stress in a shaft under torsional strain is in the nature of a shear at each side of a plane cutting across it at any point.

The shearing stress in a round shaft varies from 0 at the center to a maximum at the circumference.

If RT = the safe torsional moment of resistance of a shaft; then RT should equal $T.M.$ or $RT = T.M.$; but RT must equal the permissible working stress S of the material in pounds per square inch multiplied by Z_t, the section modulus for torsion; a constant determined by the shape of the cross-section of the shaft.

Now, as $RT = S \times Z_t$ so also must

$$W \times L = S \times Z_t$$

The permissible unit working stress S may be taken as 9,000 lbs. per square inch for steel.

The section modulus for torsion or the constant Z_t for a

$$\text{Circle} = \frac{3.1416d^3}{16} = 0.196d^3$$

$$\text{Square} = \frac{2a^3}{9} = 0.22\ a^3 \text{ approx.}$$

$$\text{Hollow Circle} = \frac{3.1416}{16}\left(\frac{d^4 - d_1{}^4}{d}\right) = 0.196\left(\frac{d^4 - d_1{}^4}{d}\right)$$

(See Fig. B)

Example. Required the diameter of a solid shaft—provided with a lever arm 15'' long, from the end of which

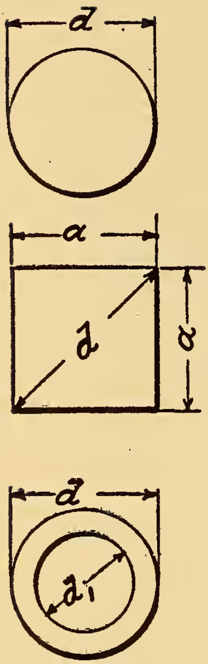


FIG. B.

is suspended a weight of 5,000 lbs. The shaft to be of steel?

Then $T.M. = W \times L$ or $15 \times 5{,}000 = 75{,}000$ inch-lbs., and as RT must equal 75,000 inch-lbs.

then also $S \times Z_t = 75{,}000.$

Now, by substitution we have

$$9{,}000 \times 0.196d^3 = 75{,}000$$

$$1{,}764.000d^3 = 75{,}000$$

$$\text{and } d^3 = \frac{75{,}000}{1{,}764} = 42.517$$

$$d = \sqrt[3]{42.517} = 3.5'' \text{ approx.}$$

For further study, see standard hand and pocket books already referred to.

PIPES AND HOLLOW CYLINDERS

These are used for storing or conveying fluids for many varied purposes. The materials used most for this purpose are cast iron, wrought iron and steel. In any case the one best suited for the purpose is selected. Small pipes for conveying water, gas, air, etc., generally are of wrought iron or steel.

The larger pipes for gas and water and the cylinders of engines and pumps are usually of cast iron.

Tanks usually are built up from wrought iron or steel plates riveted together at the joints.

For steam the pipe lengths are long, narrow plates or sheets of wrought iron or steel rolled into a tubular form, after which the two edges are welded together.

It will be readily understood that the thickness of pipes and cylinders must be such that they will withstand the internal pressure to which they are to be subjected. Cast-iron, wrought-iron and steel pipes are made up by manufacturers in standard diameters and of standard thicknesses of metal. The walls of cast-iron pipe are usually made thicker than actually required to withstand the normal pressure, in order to withstand shocks in handling and water hammer after being placed in service. Both call for large factors of safety and consequent low allowable working stresses.

It has been previously shown that fluid pressure acts in all directions and tends to tear the containing vessel apart. The pressure is usually expressed in pounds per square inch for each square inch of surface. In a pipe filled with water subjected to pressure the greatest strain tending to tear the pipe along the side lengthwise for any one inch equals the product of the diameter in inches times one inch length times the water pressure in pounds per square inch. As water is practically incompressible, it will be readily understood that for any one inch length of the pipe (see Fig. 2) the water contained on opposite sides of any one diameter is subjected to a spreading action equal to the pressure per square inch on each inch of said diameter. The spreading action is resisted by the two thicknesses of metal, one at each end of the diameter and therefore each side of the pipe resists one-half the strain and is consequently stressed to that extent.

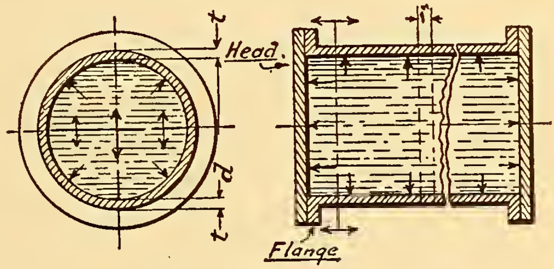


FIG. 2.

Hence the formula, Pressure (P) $\times$ diameter (d) $\times$ length (l) $=2\times$thickness (t) $\times$length (l) $\times$ultimate tensile strength of the material (S) $\div$factor of safety (f), or

$$Pdl=\frac{2tlS}{f} \text{ and } Pd=\frac{2tS}{f}$$

Example. What thickness of cast iron is required for a water pipe 10'' in diameter under a pressure of 100 lbs. per square inch, with a factor of safety of 16?

Solution. By substituting in the above equation

$$100\times 10=\frac{2\times t\times 20{,}000}{16}$$

$$\text{or } 1{,}000=2{,}500\times t, \text{ and } t=0.4''$$

If in the above case the ends of the pipe were provided with flanges, to which flanged covers are to be bolted, the pressure would tend to burst the covers and to tear the pipe apart circumferentially. In order to ascertain whether the thickness 0.4'' as obtained is sufficient, also under this condition it would be advisable to find what factor of safety this thickness would represent. If this is found to be equal to or greater than the factor (16) used above, the walls would be considered sufficiently strong.

This can readily be found by dividing the product o the net area of metal$\times$ultimate strength (S) by the total pressure exerted against the cover.

Formula. Pressure per sq. in. (P) $\times$inside diam.^{2} (d^2) $\times$ .7854 = [outside diam.^{2}(D^2) $-$ inside diam.^{2}(d^2)] $\times$ .7854 $\times$ ultimate tensile strength (S) $\div$ factor of safety (f), or

$$P\times d^2\times .7854=\frac{.7854(D^2-d^2)\times S}{f}$$

By substitution, then $100\times 10\times 10\times .7854=$

$$\frac{.7854(116.64-100)\times 20{,}000}{f} \text{ and } f=\frac{.7854\times 16.64\times 20{,}000}{10{,}000\times .7854}$$

$=33.28$

The factor as found is more than twice that assumed as necessary for the walls of the pipe; the walls, therefore, would be considered as of ample thickness.

The difference in area between the outer diameter circle and the area of the inner diameter circle represents the area of metal for resisting the strain.

The formula for all practical purposes, if the walls are relatively thin as compared to the diameter, may be expressed in the simple form:

$$P \times d = \frac{4 \times t \times S}{f} \quad \text{or}$$

$$f = \frac{4 \times 0.4 \times 20{,}000}{1000} = 32.0$$

for the example as given above.

This formula holds good also for thin hollow spheres, the inside diameter being taken as the counterpart of the inside diameter of the pipe.

RIVETED JOINTS

Rivets when used to join two plates which tend to slide one upon the other or to be pulled apart lengthwise are subjected to compression, tending to crush them where they pass through the individual plates and to shear or cutting across at the point where they pass from the one plate into the other.

Bearing Value of Rivets.—The bearing value is usually taken as equal to the rivet diameter multiplied by the thickness of the plate multiplied by the safe bearing strength of the material of the plate or rivet, depending which has the lower value or compressive strength.

Rivet Shear.—The shearing strength of the rivet is taken as equal to the area (diam.$^2 \times .7854$) of the rivet

multiplied by the safe shearing strength of the rivet material.

One of three types of riveted joints is generally used in practice, namely:

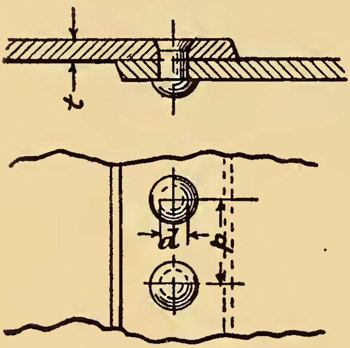


FIG. 3.

1st—Single riveted lap joint (see Fig. 3) in which the edge of one plate laps the edge of the other plate, with a single row of rivets fastening the two plates together.

Here each rivet is required to resist the tensile strain of the plate for a width equal to the center to center distance or pitch of the rivets.

If P equals the strain transmitted from the plate to each rivet as above, t the thickness of the plate, d the diameter of the rivet, p the pitch of the rivets or center to center distance between them, S_t the tensile stress, S_b the bearing value and S_s the shearing stress, each per square inch, respectively, for plates and rivets for the narrow strip represented by the pitch of the rivet; then the resultant stresses produced in single riveted joints would be:

TENSILE STRESS, (S_t) = strain (P) divided by the difference between the products of the plate thickness (t) times the pitch (p) and the thickness (t) times the diameter (d). Then the unit tensile stress in the plate,

$$S_t = \frac{P}{tp - td} \quad \text{or} \quad S_t = \frac{P}{t(p-d)}$$

BEARING, S_b = strain, divided by the thickness times the diameter of the rivet; equals the unit compressive stress of the plate and rivet, or

$$S_b = \frac{P}{dt}$$

SHEARING STRESS, S_s = strain, divided by the area of the rivet ($.7854\,d^2$) or

$$S_s = \frac{P}{.7854\,d^2}$$

2nd—Single Shear double riveted joints, in which two rows of rivets, staggered are used. (See Fig. 4)

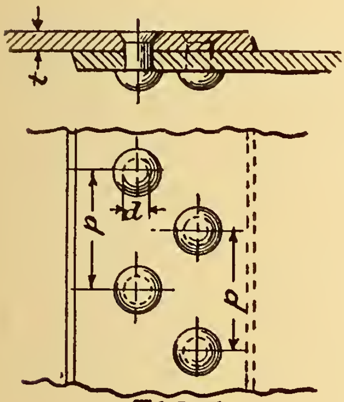


FIG. 4

The stresses in double riveted joints (single shear) are as follows:

TENSILE STRESS, this is the same as for single riveted joints.

BEARING, S_b = strain, divided by twice the thickness of the plate times the diameter of the rivet, or

$$S_b = \frac{P}{2td}$$

SHEARING STRESS, $S_s = \dfrac{P}{2 \times .7854 \times d^2}$

3rd—Butt joints in double shear, in which the edges of the plates butt one against the other instead of lapping one upon the other (See Fig. 5). A joint of this kind has two cover or splice plates, one at each side and each of which laps over the abutting plates or plates to be joined. The rivets at each side of the joint pass through the two cover plates and through the plate to be joined, lying between them; that is, they pass through the three thicknesses of plate. It will be seen that the rivets are all placed in double shear or in shear between each face of the plate to be joined and the adjacent cover plate.

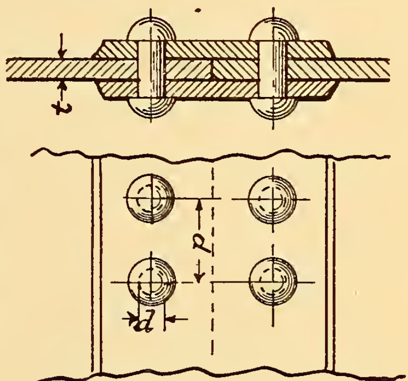


FIG. 5.

The stresses in butt joints, single riveted, double shear;

TENSILE STRESS; the same formula for single riveted lap joints is used or

$$S_t = \frac{P}{t(p-d)}$$

BEARING.—The same formula as for single riveted lap joints in like manner holds good here, also; or

$$S_b = \frac{P}{td}$$

In both cases the thickness of the joined plates is the determining factor, as the thickness of each cover plate ranges usually from 75 to 100 per cent. of the plates to be joined.

SHEARING STRESS.—The formula for the shearing stress of the rivets is, however, the same as that for a double riveted lap joint, as the plates tend to cut through the rivets at two separate points. Each rivet in double shear acting the same as two rivets in single shear, and

$$S_s = \frac{P}{2 \times .7854 \times d^2}$$

In the above joint it is true there are two rows of rivets, but each row serves only one of the joined plates, and the joint is consequently single riveted as compared to a double riveted joint; the rivets are, however, in double shear, as against single shear in single riveted joints.

Butt joints are made also as double and triple riveted for additional strength. In these cases one of the cover plates is usually wider than the other, and the outer rows of rivets are placed in single shear, as they pass through one joined plate and one cover plate only.

All holes in riveted joints should be of full bore throughout and true, as offset holes reduce the shear section of the rivet and likewise weaken the joint.

For further information on riveting and riveted joints the student is referred to the various engineers' hand books and steel manufacturers' pocketbooks.

Allowable stresses for some materials, as taken from the Building Code of Baltimore, Md., issue of 1908, are given in the following table:

Maximum Allowable Stresses for Various Building Materials. Baltimore City Building Code—1908

Material	Unit Stress or Pounds per Square Inch.				
	Tension	Compression	Shear	Bending Ext. Fibres	Bearing
Rolled Steel.........	16,000	16,000	9,000	16,000	....
Cast Steel..........	16,000	16,000	....	....	....
Wrought Iron.......	12,000	12,000	6,000	....	....
Cast Iron (Short Blocks)...........	5,000	16,000	3,000	....	....
Steel—Pins & Rivets.	....	....	10,000Shop	....	20,000
Steel—Pins & Rivets.	....	....	8,000Field	....	....
Steel Bolts..........	....	....	7,000Field	....	....
Wrt. Iron—Pins & Rivets............	....	....	7,500Shop	....	15,000
Wrt. Iron—Pins & Rivets............	....	....	6,000Field	....	....
Wrt. Iron Bolts......	....	....	5,500Field	....	....
Long Leaf Pine (with Grain)............	1,800	1,000	100	1,800	....
Long Leaf Pine (across Grain).....	....	600	500	....	....
White Pine (W. Gr.).	1,000	800	85	1,000	....
White Pine (A. Gr.)..	....	400	350	....	....
Oak (with Grain)....	1,500	1,000	100	1,500	....
Oak (across Grain)...	....	600	720	....	....
Virginia Pine(W.Gr.)	1,200	800	90	1,350	....
Virginia Pine (A.Gr.)	....	400	400	....	....

BEAMS

A long piece of timber or metal (steel, iron, etc.) supported at one or more points for carrying loads or weights and transferring the action resulting therefrom to the beam supports, the loads being off to one side and not immediately above or in line with the beam supports.

Examples. The horizontal timbers or joists and beams for supporting the floors of buildings, weighing scale beams, the walking beam of side wheel steamers, levers in machines, etc.

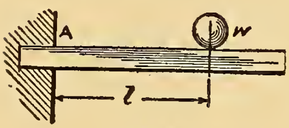


FIG. 6.

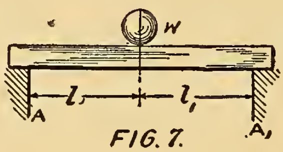


FIG. 7.

FIG. 8.

General Classes: (See Figs. 6–7–8)

A Simple Beam.—A beam fixed or supported at both ends to span the space between such supports.

A Cantilever Beam.—A beam having one end fixed in a support and the other end free.

A Continuous Beam.—A beam extending over two supports and some distance beyond each with the said distances equal. These, however, will not be considered here, as their study requires more advanced knowledge.

Reaction of the Support.—A stress exerted by the support to balance the combined force due to the weight of the beam and the load carried thereby. The weight of the beam is usually small as compared to load or weight supported thereby and is frequently neglected. It is plain the support must be strong enough to resist the force acting thereon, otherwise the structure would be destroyed. The total stress of the support or supports is equal to the load. If there is more than one support,

the nature and relative location of the load, on the basis of the theory of levers, determines what part of the load is carried by each support. In the case of the cantilever it is plain that the one support carries the entire load, including the weight of the beam. There is, however, a tendency also to topple over the support and bend the beam anywhere throughout its length, especially at the support. The extent of this tendency at any point of the beam is called the *Bending Moment* at such point.

In the case of *Cantilevers*. If W = the weight, l = the distance from the support in inches, and A = the support, and F_r = the reaction of the support; then $F_r = W$.

In the case of the *Simple Beam*. If W = weight, l = distance from the center of the weight to the left-hand support; l_1 = the distance to the right-hand support; A and A_1 the left hand and right hand supports, respectively, and likewise F, F_1 the force exerted on each one; then by considering the beam as a lever the pressure or force F, F_1 on either support may readily be found.

In considering the beam as a lever, first one support A, or A_1, is taken as the fulcrum and then the other; the weight arm being the distance from the weight to the fulcrum and the power arm the distance between the two supports. This may be expressed by the following formula:

$$F_1 \text{ (force on } A_1) = \frac{\text{Weight } (W) \times \text{distance } (l)}{\text{Length of the lever } (l + l_1)} \quad \text{or}$$

$$F_1 = \frac{W \times l}{l + l_1}$$

$$\text{And } F = \frac{W \times l_1}{l + l_1}$$

Example. Let the weight (W) equal 500 lbs. The distance 1, from one support to the weight equal 1 foot (distance expressed in inches). The distance l_1 from the weight to the other support equal 4 feet (expressed in inches); then the reaction of support A_1 equals

$$F_1 = \frac{500 \times 1' \times 12''}{5' \times 12''} = 100 \text{ lbs. reaction } A_1$$

$$\text{And } F = \frac{500 \times 4 \times 12''}{5' \times 12''} = 400 \text{ lbs. reaction } A$$

As the sum of the two reactions can be neither more nor less than the total load, then $A + A_1$ must equal 500 lbs., and is found to do so.

The distances from the weight or weights to the fulcrums, when finding the reactions only, may be expressed in feet, but in view of other formula it is considered more advisable to take them in inches also. In either case all measurements must be taken in the same unit (feet or inches).

In machines, beams in the form of levers are used extensively both as cantilevers and plain beams. The tooth of a gear outside the rim, also the crank of an engine, are cantilevers. The iceman's scales, the common seesaw, are, in fact, two cantilevers fixed rigidly together but extending from opposite sides of the support or fulcrum. A shaft extending from one hanger to another but not beyond, on which are mounted pulleys or gears, is an example of the simple beam. The pulleys tend to bend the shaft, due to their weight and the pull of the belt thereon.

If, in the example of the 500-pound load on the 5-foot length beam, the point of application of the load had been taken as the fulcrum of a lever and under a force

of 500 pounds tending to move it, but held against movement by the action of weights or forces at the ends of the two lever arms, it is evident first that the one weight would have to balance the other and that the combined weight of the two would have to equal the force of 500 pounds tending to move the lever and weights bodily from their position and all in the same direction.

In any simple beam the greatest tendency for it to bend is at the point of greatest or maximum load. In the case referred to this would be at the point of application of the weight, whether this point had been considered as the fulcrum or as the point of application of the load.

The force tending to bend a beam at any point is known as the bending moment at such point, and the maximum bending moment is consequently at the point of application of the maximum load.

In cantilevers the fulcrum is at the point of support, and the action of the forces tending to bend the beam is, therefore, greatest at this point and is the point of maximum bending moment of the beam or cantilever. This maximum bending moment in cantilever may be expressed by a formula, by applying the principle of the lever, as follows:

Let *B.M.* (max.) represent the maximum bending moment at the point of support; *W* (the weight in pounds); and 1 (the distance from the support to the point of application of the weight, in inches); then *B. M.* (max.) $= W \times 1$, in inch-pounds.

Bending Moment.—The force acting at any point of a beam tending to rupture it by tension, pulling apart at the bottom side usually, and by compression, pushing together at the top or opposite to the side in tension. Between the two somewhere and parallel therewith there

is a plane in the beam where the fibers are neither in tension nor compression. This plane passes through a point in the cross-section of the beam called the neutral axis.

Shear.—Beams, too, are subjected to shearing strains, due to their weight and the action of the loads, which tend to cut them in two crosswise, and is, of course, greatest at the points of support; this shearing tendency must be resisted and balanced by the shearing stress of the beam, while the supports must likewise counteract the crushing action to a like amount.

Loading of Beams.—In the foregoing one method of loading has been considered. The load may consist of a single weight concentrated at one point; it may be evenly distributed over the full length of the beam or it may consist of a number of loads located at various distances from the supports. The method of loading materially affects the intensity of the bending moment, and therefore each problem must be considered under its special class.

Formulas have been evolved for finding the maximum bending moments and shears for various kinds of loading, some of which are given below—in which $B.M.$ is the bending moment; 1 the length of the span in inches; W the weight in pounds; F_s the shear; and A the support.

It should be remembered that the weight of the beam acts also as a part of the total load, and is taken as a distributed load equal to the weight of the beam, distributed over its entire length. Only the extraneous or outside load is here considered for the sake of clearness. Each load and its action are computed separately and the results added together to obtain their combined action.

Simple Loading Conditions of Beams

1st. Cantilever beam, one end fixed, with a single load at the opposite or free end.

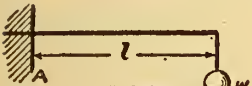


FIG. 9.

Max. $B.M. = W \times l$. At support.
Max. $F_s = W$. At support.

2nd. Cantilever beam, one end fixed, with a load evenly distributed over the full length of the beam.

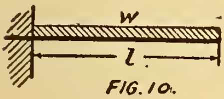


FIG. 10.

Max. $B.M. = W \times \frac{1}{2} l$. At support.
Max. $F_s = W$. At support.

3rd. Simple beam, fixed at both ends and loaded at the center.

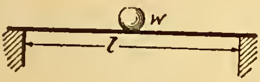


FIG. 11.

Max. $B.M. = \frac{1}{4} W \times l$. At load.
Max. $F_s = \frac{1}{2} W$. At supports.

4th. Simple beam, fixed at both ends, with a load evenly distributed over the full length of the beam.

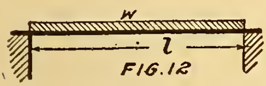


FIG. 12

Max. $B.M. = \frac{1}{8} W \times l$. At load.
Max. $F_s = \frac{1}{2} W$. At supports.

After the maximum bending moment is obtained it becomes necessary to select a beam whose safe moment of resistance R will equal or be greater than said bending moment, and whose shear resistance is greater than the shear produced by the load. The latter may be found by dividing the shear by the safe unit value allowed for shear to ascertain the required cross-sectional area necessary to resist such shear and comparing it with that of the beam whose safe moment of resistance has been found sufficient to resist the maximum bending moment.

Moment of Resistance.—The moment of resistance, R, may be determined by multiplying two factors together; one, the safe stress, f, of the material, and the other a constant, S, called the modulus of the section. This constant depends upon the form and dimensions of the cross-section of the beam.

The moment of resistance, $R = f \times S$

Section Modulus for Simple Sections or Shapes

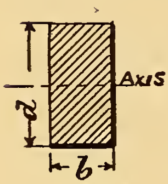


FIG. 13.

A Rectangle, where d = height in inches,
b = width in inches,
and S = section modulus;
then $S = \dfrac{b \times d^2}{6}$

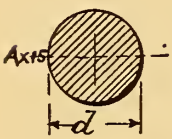


FIG. 14.

A Circle $S = \dfrac{3.1416 \times d^3}{32}$

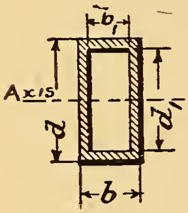


FIG. 15.

A Hollow Rectangle $S = \dfrac{b \times d^3 - b_1 \times d_1^3}{6 \times d}$

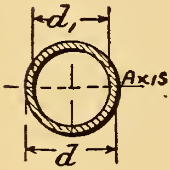


FIG. 16.

A Hollow Circle $S = \dfrac{3.1416 \times (d^4 - d_1^4)}{32 \times d}$

Example. A plain rectangular bar of steel is to support two loads, one of 600 lbs. concentrated at the center of the span (Case 3), and the other of 1,200 lbs. evenly dis-

tributed over the bar (Case 4). The span is 4 feet. What size of bar will be required to support the weight safely if 16,000 lbs. be allowed for the stress in tension, and 16,000 lbs. for compression and 10,000 lbs. for shear?

Solution. *B.M.* (max.) for the 600 lbs. weight equals

$$\frac{600 \times 48 \text{ in.}}{4} = 7{,}200 \text{ inch-pounds,}$$

B.M. (max.) for the 1,200 lbs. weight equals

$$\frac{1{,}200 \times 48 \text{ in.}}{8} = 7{,}200 \text{ inch-pounds.}$$

Combined max. bend. moment $= 7{,}200 + 7{,}200 = 14{,}400$ inch-pounds.

Now, as the moment of resistance (R) must equal the max. *B.M.* and $R = f \times S$, then also must $f \times S =$ max. *B.M.*, but the *B.M.* in this particular case is 14,400 lbs.

Therefore, $14{,}400 = f \times S$; but $f = 16{,}000$, then

$$14{,}400 = 16{,}000 \times S, \text{ and } S = \frac{14{,}400}{16{,}000} \text{ or } 0.9$$

Now $S = \dfrac{b \times d^2}{6}$, and by substitution and clearing the fraction $b \times d^2 = 0.9 \times 6$ or 5.4.

If, now, the thickness is assumed as ⅝ in. or .625″, then $.625\ d^2 = 5.4$, and $d^2 = 5.4 \div .625$ or 8.64, and $d = \sqrt{8.64}$ or approximately 2.94 inches. The nearest standard bar would be ⅝″ × 3″ or ⅝″ thick by 3″ wide.

The shear at each support is equal to ½ the total load of $600 + 1{,}200$ or 1,800 lbs. $\div 2 = 900$ lbs. Which is less than 1–10, the allowable stress in shear.

The usual practice is to use rolled shapes, such as *I*-beams, as they require, due to their shape and design, less metal and have greater stability relatively than plates or bars on edge for two reasons, viz:

1st. The greater portion or bulk of the metal is placed in the flanges and furthest away from the neutral axis, or where it will add most to the strength of the beam. The beam is made high, relative to the width of the flanges, for the same reason that a thin board on edge is stiffer and stronger than when placed or laid flat.

2nd. The flanges, even though they are much narrower than the height of the beam, are approximately eight times the thickness of the web and, therefore, afford a good bearing.

In the case of the bar for supporting the 600 and 1,200 pound loads already considered the $\frac{5}{8}'' \times 3''$ bar as found would safely sustain the load placed upon it, but might deflect or sag too much. The beams and joists in buildings for supporting floors and plastered ceilings should be stiff and should not sag or spring but a small fraction of an inch when loaded, as the sagging, springing or vibration would cause the plastered ceiling to crack.

Formula for determining the deflection (D) of beams of various shapes may be found in the various hand books published by the large steel manufacturers. They take into account the loads, their method of application and certain properties of the shape of the beam called the moment of inertia, and also the coefficient or modulus of elasticity of the material of the beam.

The *formula* for deflection (D) of a simple beam evenly loaded and rectangular in cross-section and used on edge may be expressed as follows: if W = the weight in pounds; l the span in inches; E the coef. of elasticity, which for steel is taken as 29,000,000; I the moment of inertia, based on the kind of cross-section of the beam; b the thickness of the beam in inches; and the figures 5, 384, and 12 as numerical constants.

$$D \text{ (Max. Defl. in In.)} = \frac{5 \times W \times 1^3}{384 \times E \times I};$$

$$\text{in which } I = \frac{b \times d^3}{12}$$

The value of I (moment of inertia) is found first and then substituted in the other formula in the place of I.

The student may wonder why one formula is not made to cover all cases and thereby simplify the work. This is hardly possible when the many forms and combinations of loading and shapes of sections are considered.

The hand books already referred to are for the purpose of assisting the designer and to facilitate his work. In the books not only are the various formulas given but also tables showing the values or properties of the many shapes and tables showing the carrying capacity of beams of different materials and for different lengths of span.

The student is earnestly advised to familiarize himself with the use of such pocketbooks and the data contained therein. Limited space has permitted of explaining only the meaning and application and the reason for the various terms and formulas as given in the hand books.

COLUMNS OR STRUTS

In a machine or structure such parts or members acting endwise to transfer a direct compression load from one point to another, such as the columns for supporting the floor beams and joists and the roof trusses of buildings, the piston and connecting rods of engines, the supports or legs of a machine, a table, etc. Columns and struts are the opposite to ties, which act in tension to prevent pulling apart, while columns and struts act in compression against crushing. In some machines, as in steam engines, the piston rod and connecting rod act alter-

nately in tension and compression as the piston moves backward and forward in its operation.

The strength of columns depend upon the nature of the material; the cross-sectional area; the way in which the material is distributed about the neutral axis; the length of the column as compared to its type of cross-section; the strength of the material and the nature of the ends, which latter influence the strength of the column to a very great extent.

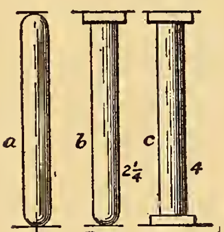


FIG. 17.

The ends may be round (see Fig. 17), as shown at *a*, or one end may be round and the other flat as at *b*, or both ends may be flat as at *c*. The form *c* is most used as in buildings and supports for machines, and is the strongest. Form *b* and form *a* are used in compression members of bridge trusses and machine parts, connecting rods, etc.

Relatively, a column with ends of type *b* is $2\frac{1}{4}$ times, and *c* is 4 times as strong as with type *a*; further, it has been found that three columns of the same cross-section, and with ends respectively as *a*, *b* and *c*, are of the same strength if their lengths vary as 1 to $1\frac{1}{2}$ and 2. Column *a* with the rounded ends is the shortest. (See Fig. 18.)

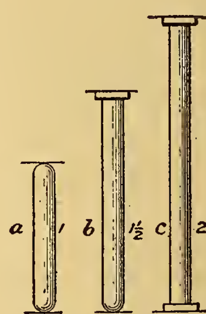


FIG. 18.

Columns are made of quite a number of different kinds of material, but the more commonly used are timber, cast iron and steel. When of timber they consist usually of one solid piece; when of cast iron they are in one piece and cast hollow, except when acting as an integral part of the frame of machines, where the cross-section may be of almost any form to meet conditions; when made of

steel, as in engines and machines, for connecting rods, piston rods, etc., they are usually solid and forged; steel pipe is often used for columns. In buildings and structures, however, the columns consist either of a single-rolled steel shape or section or a combination of such shapes or sections, either riveted directly together or held in definite relationship to each other by web plates of by lattice or diagonal bracing riveted to the main sections. In the case of the web plates they also add considerable strength in one direction.

Columns made up of more than a single-rolled shape are known as "built-up columns." The main advantages of built-up columns are:

1st. The metal may be so placed that the greater portion will be located away from or distant from the neutral axis.

2nd. To provide greater and more stable end bearings, and so distribute the weight over a greater surface.

The formulas in general use for strength of columns are empirical, and are based on the assumption that columns may fail by direct compression, by bending and compression combined, or by bending alone. The formulas agree practically with the results obtained by experimenting with columns of different kinds.

In the hand books, to which attention has already been directed, will also be found formulas for finding the moment of inertia, radius of gyration, area, etc., called properties or elements, appertaining to the various shapes (steel) and the conditions under which the shapes are strained and exert stress.

Moment of Inertia.—This is based on the area of the material of the section, the location of the neutral axis to the center of gravity of the section and the shape of the section. The neutral axis in built-up columns does not

necessarily pass through the center of gravity of the sections (individual) of the column, but through the center of the area of the column, and in such the two axes are taken at right angles to each other. Columns due to their shape in cross-section are usually more rigid laterally in one direction than in a direction at right angles thereto, because the distance from the neutral axis to the extreme fibres or outside edges in one direction is greater than the distance from the other neutral axis to the extreme fibers. This is evident when using a thin board as a strut or column. The strength of a column, therefore, is in a large measure determined by its least width of cross-section.

It is also evident from the above that there is a moment of inertia for each neutral axis.

Radius of Gyration.—This term is used to express the relation between any moment of inertia and the area of a shape, section or column under consideration. Its use is to facilitate finding the safe load any shape or section will sustain when used as a strut or column.

If r = radius of gyration, I = moment of inertia and A = area, the inch being the unit of measurement, then

$$r=\sqrt{\frac{I}{A}}$$

For a rectangular column section with the neutral axis through the center of gravity:

$$I \text{ (moment of inertia)} = \frac{b\times d^3}{12}, \qquad A \text{ (area)} = b\times d$$

and as $r=\sqrt{\frac{I}{A}}$, then $r=\sqrt{\frac{b\times d^3}{12}\div b\times d}$ or

$r=\sqrt{\frac{b\times d^3}{12\times b\times d}}$ or $\sqrt{\frac{d^2}{12}}$, from which follows;

$$r=\frac{d}{\sqrt{12}} \text{ or } 0.288675\times d$$

Ratio of Slenderness of columns equals the unbraced length l, in inches between the lateral supports, divided by the radius of gyration r. This ratio, $\frac{l}{r}$, is used to find the allowable unit fibre stress f, for the material.

For steel columns, the American Bridge Co. give the formula for allowable fibre stress, $f = 19{,}000 - 100\frac{l}{r}$. It must not exceed a maximum of 13,000. In no case should the ratio exceed 200.

Example: Let $l = 240''$ and $r = 2$, then $f = 19{,}000 - 100\left(\frac{240}{2}\right) = 7{,}000$.

The Baltimore City Code permits a maximum ratio of 120 with r, as the least radius of gyration.